History of Science Library
Editor: Michael A. Hoskin
Lecturer in the History of Science, Cambridge University

THE ENGLISH PARACELSIANS
Allen G. Debus
Professor of the History of Science, University of Chicago

WILLIAM HERSCHEL AND THE CONSTRUCTION OF THE HEAVENS
M. A. Hoskin
Lecturer in the History of Science, Cambridge University

A HISTORY OF THE THEORIES OF RAIN
W. E. Knowles Middleton

THE ORIGINS OF CHEMISTRY
R. P. Multhauf
Director of the Museum of History and Technology at the Smithsonian Institution

THEORIES OF LIGHT FROM DESCARTES TO NEWTON
A. I. Sabra
Reader in the History of the Classical Tradition, The Warburg Institute

MEDICINE IN MEDIEVAL ENGLAND
C. H. Talbot
Research Fellow at the Wellcome Institute of the History of Medicine, London

THE EARTH IN DECAY
A HISTORY OF BRITISH GEOMORPHOLOGY, 1578–1878
G. L. Davies
Lecturer in the Department of Geography, Trinity College, University of Dublin

THE CONFLICT BETWEEN ATOMISM AND CONSERVATION THEORY 1644–1860
W. L. Scott
Professorial Lecturer, School of Business Administration and Department of Mathematics and Science, American University

THE ROAD TO MEDICAL ENLIGHTENMENT 1650–1695
L. S. King
Professorial Lecturer in the History of Medicine, University of Chicago

THE SCIENTIFIC ORIGINS OF NATIONAL SOCIALISM:
SOCIAL DARWINISM IN ERNST HAECKEL AND THE GERMAN MONIST LEAGUE
Daniel Gasman
Assistant Professor of History, John Jay College, City University of New York

WILLIAM HARVEY AND THE CIRCULATION OF THE BLOOD
Gweneth Whitteridge
Lecturer in the History of Medicine, Edinburgh University

History of Science Library: Primary Sources

VEGETABLE STATICKS
Stephen Hales
Foreword by M. A. Hoskin
Lecturer in the History of Science, Cambridge University

SCIENCE AND EDUCATION IN THE SEVENTEENTH CENTURY
THE WEBSTER-WARD DEBATE
Allen G. Debus
Professor of the History of Science, University of Chicago

FUNDAMENTA MEDICINAE
Friedrich Hoffmann
Translated and introduced by Lester S. King
Professorial Lecturer in the History of Medicine, University of Chicago

THE PNEUMATICS OF HERO OF ALEXANDRIA
Introduced by Marie Boas Hall
Reader in History of Science and Technology, Imperial College, London

FORCE IN NEWTON'S PHYSICS

THE SCIENCE OF DYNAMICS IN THE SEVENTEENTH CENTURY

RICHARD S. WESTFALL
Professor of the History of Science,
Indiana University

Science History Publications
a division of
Neale Watson Academic Publications, Inc.
156 Fifth Avenue, New York 10010

First published 1971

Sole distributors for the United States and Dependencies
American Elsevier Publishing Company, Inc.
52 Vanderbilt Avenue
New York, N.Y. 10017

Sole distributors for the British Isles and Commonwealth
Macdonald & Co. (Publishers) Ltd
49–50 Poland Street
London W1A 2LG

All remaining areas
Elsevier Publishing Company
P.O. Box 211
Jan van Galenstraat 335
Amsterdam
The Netherlands

Library of Congress Catalog Card Number 76–139572
Standard Book Numbers
British SBN 356 02261 7
American SBN 444 19611 0

Printed in Great Britain by Hazell Watson & Viney Ltd
Aylesbury, Bucks

To my mother, Dorothy M. Westfall, and the memory of my father, Alfred R. Westfall.

Societas nostra id cum primis nunc agit, ut Natura et leges motus penitius, quam hucusque factum, vestigentur et innotescant. . . . Cum ignorato motu ignoretur Natura, eo diligentius scrutinio ejus Philosophis est incumbendum . . .

Our Society is now particularly busy in investigating and understanding Nature and the laws of motion more thoroughly than has been done heretofore. . . . Since Nature will remain unknown so long as motion remains unknown, diligent examination of it is the more incumbent upon philosophers . . .

Henry Oldenburg to Hieronymo Lobo,
27 May 1669

Contents

	Preface	ix
I.	*Galileo and the New Science of Mechanics*	1
II.	*Descartes and the Mechanical Philosophy*	56
III.	*Mechanics in the Mid-Century*	99
IV.	*Christiaan Huygens' Kinematics*	146
V.	*The Science of Mechanics in the Late Seventeenth Century*	194
VI.	*Leibnizian Dynamics*	283
VII.	*Newton and the Concept of Force*	323
VIII.	*Newtonian Dynamics*	424
	Appendices	526
	Bibliography	551
	Index	567

Preface

MORE years ago than I care now to recall, this book began as a study of Newton's concept of force, and the title, *Force in Newton's Physics*, the suggestion of Michael Hoskin, Editor of the History of Science Library, was tentatively bestowed on the project at that time. Historian that I am, I understood from the beginning that any examination of Newton's concept of force worthy of the name would need to see it in the setting of seventeenth-century mechanics as a whole, but in my innocence I scarcely realised what such a project would involve. I was still primarily at work on Newton himself when the invitation to deliver a paper on Galileo at UCLA's quadricentennial celebration impelled me to examine Galileo's mechanics anew from the point of view of the concept of force. The revelation of how much the seventeenth century before Newton could teach me in illuminating his dynamics set the pattern for the rest of my investigation. To the degree that Newton still occupies more than a quarter of the space, the work maintains its original intent. To a considerable extent, however, it has also become a history of dynamics in the seventeenth century, and I have acknowledged as much by adding the subtitle that the book now bears.

If the study is a history of dynamics in the seventeenth century, I like to think that it is a history with a difference. Although I explore all of the major advances in dynamics in some detail, I have not concerned myself primarily to catalogue them. Rather I have tried to understand the obstacles that cluttered the path leading toward modern dynamics, or to employ a different figure, the conceptual knots that had to be loosed. It would be correct to say that I have tried to see seventeenth-century dynamics through the eyes of the men engaged in creating the science – I have attempted to define the problems on which they expended themselves in their terms, and to see their proposed solutions in relation to the intellectual equipment at their disposal. Equally, however, I have sought to exploit the advantages that three centuries of perspective provide. I do not believe that any scientist of the seventeenth century could have stated the set of questions around which this study revolves with the clarity possible for the historian of the twentieth century. If it is necessary to see dynamics through their eyes in order not merely to recount those achievements that still seem correct to us, so also it is necessary to examine their confusions from our distance in order fully to understand what their problems were. I began the study of seventeenth-century dynamics predisposed to be impressed, and I have not been disappointed. Basic dynamics has now been rationalised and systematised to the extent that an intelligent schoolboy can master all that the seventeenth century produced in a few weeks. A careful study of how they produced it is calculated to remove any suspicion that the task was, after all, rather simple. Seen from the dual perspective I have proposed, the creation of modern dynamics in the seventeenth century appears as one of the supreme conquests of the human spirit, a triumph wholly worthy of the 'century of genius'.

Should any doubt remain, let me say straight out that this is a history of ideas. I have paid almost no attention to the social and economic setting in which dynamics emerged. When the book closes, the bourgeoisie have not risen through my efforts one inch above the level they occupied on page one. Undoubtedly the fact that many men of ability devoted serious study to natural science in general and mechanics in particular during the seventeenth century depended to a high degree on the social and economic state of Europe. If my experience is any guide, however, it is impossible to conclude from seventeenth-century literature on mechanics that practical considerations, technological problems set by the economic system, guided and determined the conceptual development of the science. The most important applications of dynamics at the time were to problems in pure science – the celestial dynamics of Newton's *Principia* epitomised the uses to which it was put – and as far as the seventeenth century was concerned, it was more by accident than design that the engineers of a later age would exploit its conclusions to such effect. The story I have to tell, then, concerns itself, not with social contexts or with practical considerations, but with conceptual developments initiated by a new idea of motion proposed near the beginning of the century. Whereas I devote no attention to social factors, I devote very little more to technical mathematical questions. I do not mean to deny in any way the importance of mathematics in seventeenth-century dynamics. With the calculus, for example, a whole new range of problems hitherto beyond the grasp of quantitative mechanics became amenable to exact treatment. My central concern has focused on conceptual issues, however; and during the development of dynamics up to Newton, such matters appear to me to have been central to the science of dynamics. My attempt has been to follow the conceptual development within the limits imposed by the body of inherited ideas on the one hand, and on the other by the prevailing philosophy of nature.

Somewhat to my surprise, I realised as I was completing the manuscript that I was perhaps commenting implicitly on a question now exercising the discipline of the history of science. Is scientific change a matter of gradual development or of sudden paradigm shift? Let me only state, as my text will amply document, that the study of seventeenth-century dynamics has impressed me with the incredible tenacity of received ideas in a conceptual context that appears antithetical to them from a distance of three centuries. Looking back, we want to find Newtonian dynamics in Galileo. In fact, nearly a century's labour by a host of able men was required to extract from the new conception of motion consequences that appear obvious to us. In the meantime, a set of ideas descended from medieval and ancient mechanics, ideas that continued to appear familiar and therefore valid, despite the new conception of motion, dominated attempts to construct a quantitative dynamics. However one may interpret the change from Aristotelian dynamics, it is difficult to find evidence of a dramatic paradigm shift in the seventeenth-century literature on mechanics.

Because the change was gradual, a considerable ambiguity of technical language existed. Without exception, I believe, the technical terms of Newtonian dynamics descended from those in common use in earlier mechanics. Words that acquired a precise meaning with Newton were part of the coinage in which the commerce in mechanics had long been carried on, and for want of alternate coinage, they continued in use during the seventeenth century. Those who employed them undoubtedly thought they were clear enough; in relation to the new dynamics that eventually appeared, however, they were charged with ambiguities. I must warn the reader not to impose the precise modern meaning on such words. When '*velocitas*' appeared in a Latin text, or '*gravité*' in a French one, I could do no other than to translate the words as 'velocity' and 'gravity', and I could scarcely alter an English text to replace the word 'force'. A fair part of the history I recount has to do with the development of precise terminology, and the indication of prevailing ambiguities is essential to my task. Let the reader then exercise care in the meanings he imposes. As for me, the growing realisation of the problem of terminology in seventeenth-century dynamics led to extensive essays on usage, especially of the word 'force'. These essays compose the appendices referred to in the footnotes.

In connection with usage, I need also to remark on translations. Rather than translate anew every passage I include, I have utilised standard accepted translations where they are available. Whenever specific terms have been at issue, I have checked with the original texts to be sure of the exact word or phrase.

Like every author who completes an extended work, I am fully aware of the help I have received in many forms from many sources. Without the National Science Foundation, I could never seriously have embarked on the project, and I wish to acknowledge its generous support. A grant-in-aid from the Office of Research and Advanced Studies of Indiana University facilitated the preparation of the final manuscript. Most of the research was conducted in three libraries – University Library Cambridge, the Widener Library at Harvard, and the library of Indiana University. To the expert assistance of the staffs in all three I extend my thanks. The Syndics of Cambridge University Library have graciously permitted me to quote extensively from the Newton papers in their collection. Members of the secretarial staff of the Department of History and Philosophy of Science at Indiana University applied skill and willingness beyond any reasonable demands of their positions in the preparation of the manuscript, and I wish to thank especially Joyce Chubatow, Jean Coppin, and Ina Mitchell. And finally I owe a debt beyond calculation to my wife Gloria and my children who long endured a husband and father frequently engrossed in matters three centuries removed. It is not at all clear that I shall return to the twentieth century now that the book is done, but it is equally unclear that I should be better company if I did.

Chapter One

Galileo and the New Science of Mechanics

In the introductory passages of the *Mathematical Principles of Natural Philosophy*, published near the end of the seventeenth century, in 1687, Isaac Newton set down the three laws of motion, which became the foundation of the science of mechanics and of the entire structure of modern physical science.

> Law I. Every body continues in its state of rest, or of uniform motion in a right line, unless it is compelled to change that state by forces impressed upon it.
>
> Law II. The change of motion is proportional to the motive force impressed; and is made in the direction of the right line in which that force is impressed.
>
> Law III. To every action there is always opposed an equal reaction: or, the mutual actions of two bodies upon each other are always equal, and directed to contrary parts.[1]

As far as the third law was concerned, Newton believed that its formulation was original with him – although, of course, he also believed that the phenomena cry out for it. The first two laws he attributed to Galileo as a matter of common knowledge.

> By the first two Laws and the first two Corollaries, Galileo discovered that the descent of bodies varied as the square of the time and that the motion of projectiles was in the curve of a parabola . . .[2]

In this book, I am primarily concerned with the second law, the concept of force and its role in rational mechanics. Inevitably I find Newton's attribution suggestive.[3] Early in the seventeenth century, Galileo transformed the science of mechanics and introduced the problem of force in its present form. His mechanics and his philosophy of nature seem to demand the second law of motion – so much so that more than one historian of science has followed Newton by implicitly ascribing it to him. A handful of crabbed precisionists excepted, everyone agrees that Galileo contributed decisively to the first law, even though he did not formulate it exactly. The first law in turn demands the second, to the point that we can hardly imagine them apart. Ernst Mach wondered why Newton had bothered to state the first since it is implicit in the second. Fortunately for Newton's role in the history of physics, he gave Galileo more than his due. However suggestive we may find Galileo's mechanics, he did not formulate the second law of motion. Its formulation was Newton's work and not the least of his claims to immortality.

It was not solely Newton's work, however, and Galileo cannot be neglected in the history of the concept of force. Indeed, Galileo looking forward early in the century was better as a prophet than Newton was as an historian looking back near its end. There 'have been opened up to this vast and most excellent science, of which my work is merely the beginning, ways and means by which other minds more acute than mine will explore its remote corners.'[4] Except for the suggestion of other minds more acute (which we may understand as a rhetorical flourish, and not as a serious expectation on Galileo's part), his words forecast the course of mechanics in the seventeenth century. Newton's *Principia* was the culmination of more than half a century's labour, which took its start from Galileo. After the new conception of motion, to which Galileo's contribution was basic, the second law was the major achievement of seventeenth-century mechanics. Galileo yoked the new conception of motion to a new ideal of mechanics which set as its goal the mathematical description of the phenomena of motion. Until the formulation of the second law, the new ideal remained more hope than fact. Realised at last in the *Principia*, it became at once the basis and the example of mathematised science.

Galileo's initial work in mechanics predated the seventeenth century by about a decade. During the period 1589–92, when as a young man he occupied the chair of mathematics at the University of Pisa, he composed an essay, *De motu*, which remained unpublished until the nineteenth century; and in 1593, shortly after he had moved to the University of Padua, a treatise on the simple machines displayed

another facet of his interest. Together, the two works provide the basic evidence of Galileo's early understanding of mechanics. Beginning in the first decade of the seventeenth century while he was still at Padua, Galileo fundamentally revised his earlier views, partly under the stimulus of his commitment to Copernican astronomy, the justification of which entailed a new conception of motion. As a result of the fame that his telescope brought, he was able to return to Tuscany and Florence in 1610 with the grandiloquent title of Chief Mathematician and Philosopher to the Grand Duke, and in Florence he completed the two books that accomplished a revolution in the science of mechanics – *Dialogo sopra i due massimi sistemi del mondo* (the *Dialogue*, 1632) and *Discorsi e dimostrazioni matematiche intorno a due nuove scienze* (the *Discourses*, 1638). Galileo died in 1642, blind and confined to his home as a prisoner following his trial before the Inquisition in 1633 for his defence of Copernican astronomy in the *Dialogue*. His *Discourses* had to be spirited abroad in manuscript and published in the Netherlands.

Since Galileo's mechanics transposed the problem of force by placing it in a new context, the history of the second law of motion effectively begins with him. The Aristotelian and medieval science of mechanics had not recognised any effective distinction between statics and dynamics. The simple machines, such as the lever, obviously served, not to hold bodies in equilibrium, but to move them; and in the Aristotelian tradition, the simple machines were analysed in dynamic terms. The most sophisticated conception of force in medieval mechanics is found in the treatises on statics ascribed to Jordanus. What we call dynamics, the consideration of motions not constrained by simple machines, was built on principles directly transposed from statics and justified by the basic proposition of peripatetic mechanics, that *omne quod movetur ab alio movetur* (everything that is moved is moved by something else). Aristotle's analysis of motion in the *Physics* reveals its full meaning when we imagine the two bodies to be placed on the ends of a lever.

> If, then, A is the moving agent, B the mobile, C the distance traversed and D the time taken, then A will move $\frac{1}{2}B$ over the distance $2C$ in time D, and A will move $\frac{1}{2}B$ over the distance C in time $\frac{1}{2}D$; for so the proportion will be observed. Again, if A will move B over distance C in time D and A will move B over distance $\frac{1}{2}C$ in time $\frac{1}{2}D$, then E $(=\frac{1}{2}A)$ will move F $(=\frac{1}{2}B)$ over distance C in time D; for

> the relation of the force $E(\frac{1}{2}A)$ to the load $F(\frac{1}{2}B)$ in the last proposition is the same as the relation of the force A to the load B in the first, and accordingly the same distance (C) will be covered in the same time (D).[5]

With Galileo's denial that every body in motion requires the continued action of a mover, the concept of force began a new career which culminated in Newton's second law.

Two major achievements of Galileo's mechanics, worked out gradually during the first three or four decades of the seventeenth century and made accessible to the scientific world in his two great books, the *Dialogue* and the *Discourses*, immediately strike the modern reader. The first is the new conception whereby motion (at least horizontal motion) is held to be, not a process requiring a cause, but a state to which a body is indifferent, a state in which it remains, as it remains in rest, until some external agent causes it to leave. Once motion on a frictionless horizontal plane is acquired, Galileo declared, 'it will continue perpetually with uniform velocity.'[6] Only recently have we been taught to ask what Galileo meant by a horizontal plane. Since a horizontal plane was one equally removed at every point from a gravitational centre, his perpetual motion with uniform velocity took place on a spherical surface like that of the earth. We cannot then properly say that he stated the principle of inertia, which holds for rectilinear motion.

Once we have seized that limitation, we must be careful not to ignore the extent to which Galileo approached the concept of inertia. He identified uniform horizontal motion and rest, considering rest merely as a special case, what he called an infinite degree of slowness.

> Motion, in so far as it is and acts as motion, to that extent exists relatively to things that lack it; and among things which all share equally in any motion, it does not act, and is as if it did not exist.

To the relations of things that move together, motion is 'idle and inconsequential'; it is 'operative' only in relation to other bodies that lack it.[7] Since a body is indifferent to motion, it can participate freely in more than one motion at the same time. Thus by considering two perpendicular but not mutually obstructing motions, Galileo demonstrated that the trajectory of a projectile is a parabola. Small wonder that Newton mistook such a conception for his first law of motion.

Galileo's second major achievement in mechanics was the identification of free fall as a uniformly accelerated motion and the detailed exposition of its role in nature. 'A motion is said to be uniformly accelerated, when starting from rest, it acquires, during equal time-intervals, equal increments of speed.'[8] Galileo insisted that we understand the definition, not as an arbitrary one, such as a mathematician might construct in order to examine its logical consequences, but as a definition answering to natural phenomena. If we neglect the small resistance of the atmosphere, every body falls with uniformly accelerated motion, and Galileo used the phrase 'naturally accelerated motion' as fully synonymous.

> When I consider that a stone, which falls from some height starting from rest, constantly acquires new increments of velocity, why should I not believe that these additions are made in the simplest and easiest manner of all? The falling body remains the same, and so also the principle of motion. Why should the other factors not remain equally constant? You will say: the velocity then is uniform. Not at all! The facts establish that the velocity is not constant, and that the motion is not uniform. It is necessary then to place the identity, or if you prefer the uniformity and simplicity, not in the velocity but in the increments of velocity, that is, in the acceleration.[9]

The passage appears to be intimately related to the new conception of motion. Having concluded that uniform motion is a state which requires no cause, Galileo was free to identify a new dynamic product of weight. When a heavy body falls, its weight generates, not a uniform velocity, but uniform increments of velocity, that is, a uniform acceleration.

The concept of uniform acceleration was only half of Galileo's revision of the understanding of free fall. Aristotelian mechanics had set velocity proportional to weight. What would have been more natural in revising Aristotle than to set acceleration proportional to weight? Since Galileo put such an opinion into Sagredo's mouth in the *Dialogue*, we may be sure that he considered it.[10] As everyone knows, he concluded ultimately that all bodies fall toward the earth with the same acceleration, the variation produced by the resistance of the atmosphere being ignored. It is necessary, Salviati explained to Sagredo,

> to distinguish between heavy bodies in motion and the same bodies at rest. A large stone placed in a balance not only acquires additional weight by having another stone placed upon it, but even by the addition of a handful of hemp its weight is augmented six to ten ounces according

> to the quantity of hemp. But if you tie the hemp to the stone and allow them to fall freely from some height, do you believe that the hemp will press down upon the stone and thus accelerate its motion or do you think the motion will be retarded by a partial upward pressure? One always feels the pressure upon his shoulders when he prevents the motion of a load resting upon him; but if one descends just as rapidly as the load would fall how can it gravitate or press upon him? Do you not see that this would be the same as trying to strike a man with a lance when he is running away from you with a speed which is equal to, or even greater, than that with which you are following him?[11]

The dynamic effect of weight is not the generation of a uniform increase of velocity after all. It is rather the generation of a uniform increase of motion, where 'motion' is understood as the product of velocity and the size of the moving body (which is understood implicitly to be proportional to its weight). Again, it is hardly surprising that Newton saw the second law of motion in such passages.

Galileo's mechanics appears even more familiar when we see it against the backdrop of his conception of nature. He denied the anthropocentricity of medieval cosmology, according to which the whole universe exists for the benefit of man, and he denied as well the uniqueness of the earth, or of any other body, as a centre in the cosmos. He affirmed the homogeneity of matter, holding that all bodies are composed of the same material, packed more or less densely together. Matter is inert, and bodies composed of it obey the compulsion of external agents and move according to fixed laws. On a horizontal plane, every body 'finds itself in a condition of indifference as to motion or rest; has no inherent tendency to move in any direction, and offers no resistance to being set in motion.'[12] Because all matter is identical, the distinction of heavy and light in vertical motion vanishes, and all bodies, being heavy, move downward when not restrained. Galileo suggested that generation and corruption, the most fundamental changes in Aristotelian science, are nothing more than 'a simple transposition of parts . . .'[13] The well-known passage in the *Assayer* that denies the reality of many qualities appears to assert that physical nature is composed solely of particles of matter. 'I do not believe,' he stated, 'that for exciting in us tastes, odours, and sounds there are required in external bodies anything but sizes, shapes, numbers, and slow or fast movements; and I think that if ears, tongues, and noses were taken away, shapes and numbers and motions would remain but not

odours or tastes or sounds.'[14] These are familiar notions to the student of seventeenth-century science. Powerfully they evoke the mechanical philosophy of nature and render Galileo's analysis of free fall still more suggestive of the second law of motion.

Nevertheless, neither the second law of motion nor a satisfactory conception of force is to be found in Galileo's mechanics. The problem is not one of kinematics versus dynamics. Although there are extensive kinematical passages in Galileo, his mechanics rested on a consciously dynamic analysis of free fall. At the beginning of that analysis, he insisted that his definition of uniformly accelerated motion was not an arbitrary one, such as a mathematician might propose, but one that corresponded to a motion nature continually employs. He also reminded his readers frequently that motion on a horizontal plane is uniform because there is neither a tendency to increase motion nor a repugnance that works to diminish it. A crucial proposition of his mechanics, that the total change of velocity is determined by a body's vertical displacement independently of the path it follows, rested frankly on a dynamic analysis of motion on inclined planes. Far from restricting itself to kinematics, Galileo's mechanics returned to the dynamics of free fall at every critical point, and attempted to illuminate the whole of mechanics with its light. Moreover, in his analysis of free fall, weight functions exactly as force in Newton's second law, so that his analysis established the paradigm for the treatment of force in classical mechanics, the simplest case, in which a constant force produces a uniform acceleration. For all of that, neither the second law of motion nor a satisfactory conception of force can be found in Galileo's mechanics.

It is not a quibble on words in which I am interested. A problem of terminology certainly existed in seventeenth-century mechanics as a set of new concepts struggled to achieve precision. Viviani was so overborne by the verbal anarchy when he expanded a passage in the *Discourses* that he referred to '*l'impeto*, *il talento*, *l'energia*, or we might say *il momento*,' of a moving body.[15] Indeed we might say! We might also say, as Galileo said more than once, '*la virtù*,' and '*la propensione al moto*.' Among the other terms used more or less synonymously was the one that Newton chose and defined, *la forza* (in Latin, *vis*).[16] The chaos of terminology must have helped to obstruct the emergence of a concept of force, but terminology was not the decisive aspect of the problem. Galileo never suggested that the acceleration of free fall is produced by a force acting on a body. He never considered free fall as a special case of a general phenomenon, such that its analysis could be applied to the understanding of all changes of motion.

A problem of major dimensions for the history of science in the seventeenth century is involved in the second law of motion. The second law made possible the perfection of the mathematical science of mechanics, the supreme achievement of the scientific revolution. In the *Principia*, rational mechanics produced the masterpiece of seventeenth-century science. What is more important than the *Principia* itself and the law of universal gravitation, rational mechanics established the pattern on which physical science has modelled itself ever since. Galileo's mechanics proposed the paradigm of the second law in the analysis of free fall, a paradigm apparently so clear that Newton himself attributed the second law to Galileo. Obviously it was not that clear. Half a century elapsed between the *Discourses* and the *Principia*, and the labour of the age's ablest men did not suffice to produce a workable conception of force before Newton. In following the story of the concept of force, we follow the steps of seventeenth-century science as a whole and face the basic problems with which its creators grappled.

★

Undoubtedly, the very nature of Galileo's mechanics was a major cause of the difficulty the seventeenth century experienced in seeing that it implied the equivalent of the second law of motion. With Newton to guide us, we do not find it hard to discover insightful passages; the others we know how to interpret. His mechanics must have presented a different appearance to those who did not yet have Newton to guide them. Galileo himself belonged to the latter group, of course. He did not think of himself as struggling to clarify a concept of force. When we attempt to examine his mechanics from a pre-Newtonian point of view, instead of pregnant suggestions of the second law of motion, it appears to be dominated by quite a different set of ideas. If they tend toward any conception of force, it is one quite different from Newton's.

Most prominent is the idea of natural motions. Perhaps nothing in Galileo's mechanics separated him more emphatically from Newton, from whose universe natural motions had been banished. Galileo's universe, in contrast, contained two of them, and they made an appearance on nearly every page that he wrote. The uniform acceleration of free fall was to Galileo the 'natural acceleration downwards common to all bodies.'[17] Again, he declared that acceleration occurs when a body 'is approaching the point toward which it has a tendency, and retardation occurs because of its reluctance to leave and go away from that point . . .'[18] However much we want to read his analysis of free fall as the description of a uniform acceleration produced by a constant force, Galileo never treated it in such terms. Weight was not a force acting on a body to accelerate it. On the contrary, weight was more a static force which a body exercises on another that restrains it from its natural motion. The interpretation of what Galileo meant when he spoke of the natural tendency of bodies to move downward presents problems, as I shall wish to discuss later. Suffice it to say at this point that the idea of a natural tendency cannot be explained away from Galileo's mechanics since it expressed his conviction that the universe is an ordered one in which bodies have natural places. Another consequence of that conviction was his use of the word 'force' to express that which opposes the natural order.[19] Motion upward was 'forced' motion. Following a long tradition, he frequently used the words 'violence' and 'violent' in the same context. Forced motion did violence to the natural order. As a natural motion, the uniform acceleration of free fall resisted identification with other changes of motion.

If free fall was a natural motion, so also was the uniform motion of terrestrial objects around the centre of the earth. Uniformly accelerated motion toward a centre was the natural motion of bodies seeking their place; uniform motion around a centre was the natural motion of bodies that had found their place. Galileo employed the concept of (circular) inertia to solve the problems raised by Copernican astronomy. How is it possible for phenomena of motion to appear as they do if the earth is turning daily on its axis? When a ball is dropped from a tower, the motion of the tower from west to east should leave it far behind, and it should appear to fall well to the west. In fact, it appears to fall perpendicularly along the side of the tower. How is this possible on a moving earth? 'Keeping up with the earth,' Galileo explained, 'is the primordial and eternal motion ineradicably and inseparably participated in by this ball as a terrestrial object, which it has by its nature and will possess forever.'[20] Elsewhere he referred to the diurnal motion, which all terrestrial bodies share whether they are attached to the earth or not, as 'natural and eternal,' as 'a thing indelibly impressed upon them by nature,' as 'a natural propensity,' as 'an inherent and natural inclination.'[21]

If the diurnal rotation of terrestrial objects around the axis of the earth is natural, it would appear that this motion is distinct from other horizontal motions, just as natural acceleration is unique. Indeed, Galileo did make the distinction more than once, but the predominant tendency of his mechanics was to identify all uniform horizontal motions with the eternal motion of terrestrial bodies. In the *Dialogue*, Simplicio brought up the example of a ship in motion. Dropping a stone from the top of its mast should be equivalent to dropping it from a tower on a moving earth, and since he was convinced that the stone falls to the rear of the moving ship, Simplicio argued that its fall parallel to the tower proves that the earth must be at rest. In replying to Simplicio, Salviati insisted at first that the artificial motion of the ship cannot be compared to the natural motion of the earth. He quickly dropped the distinction, however, asserted that the stone falls at the foot of the mast, and explained the phenomenon in terms almost identical to those employed for the natural motion. The stone carried by the ship acquires 'an ineradicable motion' as fast as that of the ship; the ship's motion remains 'indelibly impressed' on the stone when it is dropped.[22] Another thought experiment in the *Dialogue* places two men in the cabin of a ship. They jump; they play catch; butterflies flutter about; fish swim in a bowl; water drips from a bottle; smoke rises – and everything that happens when the ship is at rest proceeds exactly the same when it is in motion because 'the ship's motion is common to all the things contained in it . . .'[23] Yet again, a piece of wax so little heavier than water that it descends only a yard in a minute is placed in a vase on a ship that moves one hundred yards a minute. To the men on board it appears to descend vertically.

> Now these things take place in motion which is not natural, and in materials with which we can experiment also in a state of rest or moving in the opposite direction, yet we can discover no difference in the appearances, and it seems that our senses are deceived. Then what can we be expected to detect as to the earth, which, whether it is in motion or at rest, has always been in the same state? And when is it that we are supposed to test by experiment whether there is any difference to be discovered among these events of local motion in their different states of motion and of rest, if the earth remains forever in one or the other of these two states?[24]

Whenever he was concerned with problems analogous to the rotation of the earth – problems which he raised in order to explain how the rotation of the earth can be reconciled to the observed phenomena of motion – Galileo emphasised the ultimate identity of all uniform horizontal motions, natural and artificial. For the sake of simplicity, we can call such motion inertial motion, though in using the phrase we must remember that inertial motion to Galileo was a circular motion.

Another class of problems in Galileo's mechanics introduced motions which Galileo treated in different terms although we consider them indistinguishable from inertial motion. The words '*momento*' and '*impeto*', used interchangeably, frequently designated a body's motion. In Galileo's system, the *momento* or *impeto* of a body cannot be lost in horizontal motion from which all obstacles have been removed. It can also not be generated. Motion downward generates velocity; motion upward destroys velocity; motion on a horizontal plane conserves velocity. Hence in Galileo's own terms, the *momento* or *impeto* of a body at any time should only express its inertial motion, the state which can be perceived only by a body's relation to others that do not share it, the state to which it is indifferent. Unfortunately, the verbs that Galileo used with *momento* and *impeto* suggest anything except indifference. The *impeto* which the bob of a pendulum acquires in descending with a natural motion is able to drive it upward by a forced motion (*sospignere di moto violento*) through an equal ascent. In general, when a heavy body falls from any height, it acquires just as much *impeto* as was necessary to carry it (*tirarlo*) to that height. A point on the circumference of a moving wheel has the *impeto* with which to hurl (*scagliare*) a stone. When water in a barge is set in oscillation by an uneven motion, it rises at one end, falls again because of its weight, and, pushed (*promossa*) by its own *impeto*, goes beyond equilibrium. He also imagined cases in which the *impeto* acquired by one body in falling was transferred to another and functioned to drive (*cacciare*) the second upward.[25] Since the entire tradition of mechanics before Galileo had treated force as that which is opposed to the natural order, that which compels a body to move contrary to its natural inclination, a student of mechanics in the seventeenth century was more likely to see a concept of force in these passages than in Galileo's analysis of naturally accelerated motion. Such a concept of force would not only have differed from Newton's; it would have been quantitatively incompatible with it as well. I shall refer to it as the paradigm or model of impact. The fact that a notion harking back both to the medieval theory of impetus and to the medieval science of weights survived in Galileo's mechanics can help us both to comprehend the basic thrust of his mechanics and to understand the problems which he left to be solved by the seventeenth century after him. We must remember constantly that he presented, not one, but two models of force, and the one we are least prepared to recognise was the one most adapted to the comprehension of his age.

★

Inevitably the ideal of *momento* or *impeto* recalls the treatise *De motu*, Galileo's first independent effort in mechanics, which was composed sometime around 1590, while he was teaching at Pisa. To understand his mechanics and his contribution to the concept of force, we must first analyse the structure of *De motu*.

At much the time when he was composing it, Galileo jotted down a note which illuminates the purpose of the treatise and presents a theme which dominated his mechanics through all its changes. 'A fragment of Euclid asserts that heavy and light are to be handled mathematically.'[26] Galileo's treatment of motion underwent a fundamental revision after he completed *De motu*, but his mechanics never veered from the direction he took at the beginning. Although he adopted the theory of impressed force in *De motu*, he gave it a twist not to be found among its medieval exponents. By defining impressed force as accidental lightness, he endeavoured to render it measurable by the weight to which it is equal. Which is to say, the mathematical treatment of heavy and light was equivalent to the development of a mathematical science of mechanics.

Fundamental to the structure of *De motu* was the assertion, which also remained unaltered in his mature mechanics, that all bodies are heavy. What conditions are necessary, Galileo asked, for a motion to be called natural? First, it cannot be infinite and indeterminate. Those things which are moved by nature 'are carried toward some goal where they can be at rest naturally.' Second, the body moved must be moved by an intrinsic and not by an extrinsic cause. Motion downward has a goal which is definite, the centre of the earth. Motion upward, in contrast, must always have an external cause. Where is the end of up? When would a body arrive there?

> The matter of all bodies is the same, and it is heavy in all of them. But the same heaviness cannot have contrary natural inclinations. Therefore, if there is one natural inclination, the contrary inclination must be against nature. The natural inclination of heaviness, however, is toward the center. Therefore, that which is away from the center must be against nature.[27]

Since every body strives to move toward the centre of the earth, which is its natural place, it can remain at rest outside its natural place only if a force equal to its weight restrains it.

In this concept Galileo saw the means to quantify impressed force. What is the impressed force that makes a heavy body move upward? It is a taking away of heaviness, Galileo replied. As fire deprives a piece of iron of its coldness, impressing heat on it, so hurling a stone upward deprives it of heaviness and impresses lightness on it. Why call impressed force lightness? Because we say that bodies that move upward are light, and there is no assignable difference between the stone as it rises and a light body. The stone is not naturally light, of course; it is accidentally or preternaturally light because its motion upward is against nature and forced. Galileo compared it to a piece of wood, which is naturally heavy, but which rises with a preternatural motion when it has been submerged in water. As the heat in a piece of iron gradually dissipates, so the impressed force fades. When it equals the weight of the stone, the top of the trajectory has been reached – an equation which only repeats the proposition that a body at rest outside its natural place must be restrained by a force equal to its weight. As the impressed force continues to fade, the weight of the stone preponderates, and it begins to fall.[28]

The comparison of the stone to a piece of wood in water was important. In *De motu*, Galileo employed Archimedean fluid statics to revise Aristotle's treatment of the role of the medium in motion. On the one hand, Aristotle had assigned the continuation of projectile motion to the medium. According to the basic proposition of his mechanics, a body moves only if something moves it, and in the case of a projectile separated from the original projector, the role of mover was assigned to the medium. Impetus mechanics transferred the motive force from the medium to the projectile – a body placed in motion acquires an impressed force or impetus, an internal motive power which continues the body's motion after it has separated from the projector – and Galileo's *De motu* repeated what had virtually become the accepted orthodoxy. Aristotle's mechanics also assigned a second role to the medium; it functioned as a resistance as well, and the velocity of motion was held to depend on the proportion of motive force to resistance. In the special case of a heavy body falling, Aristotle said that its velocity is directly proportional to its weight and inversely proportional to the resistance (or density) of the medium. If we commit the anachronism of putting Aristotle's analysis in functional form, we can represent it by the formula

$$v \propto \frac{F}{R}$$

where F represents the weight acting as motive force and R the resistance of the medium. Within the tradition of impetus mechanics, the contradictory nature of the two roles assigned to the medium was a well-established criticism of Aristotle's theory, and Galileo's repetition of it held nothing original.

What was original in *De motu* was his quantitative critique drawn from fluid statics. By its means, Galileo showed that Aristotle's treatment of fall leads to inconsistencies. Take bodies of two different materials, a piece of lead, say, and a piece of wood. According to the formula we used to express Aristotle's position, the proportion of their velocities (whatever the proportion may be) should be the same in all media. If their velocities in air are in the ratio of two to one, their velocities when they fall in water should be in the same ratio, although both fall more slowly. In fact, of course, wood does not fall in water at all.

To resolve the discrepancy, Galileo employed a concept of effective weight which altered the mathematical role of the medium's resistance. By the Aristotelian formula, which places the resistance in the denominator, the density of water would have to be infinite to account for the fact that wood falls through it with a speed of zero. In the case of lead, on the other hand, the density of water would be finite. To reconcile the two cases, Galileo proposed that the weight of an equal volume of the medium be subtracted from the weight of the body and not divided into it. All bodies are heavy, but some are heavier specifically than others. Why does wood rise when it is submerged in water? Because an equal volume of water has a greater weight than the wood. The wood moves up with a 'force' equal to the amount by which the weight of an equal volume of water exceeds its weight. Correspondingly, the piece of lead moves down with a 'force' equal to the amount by which its weight exceeds that of an equal volume of water.[29]

De motu proposed a revision in the basic formula of dynamics. The resistance of the medium affects the speed of a falling body by subtracting from the maximum speed, in effect, instead of dividing into it. The suggestion was not original with Galileo; it had a long history stretching back at least to John Philoponus in the sixth century. Even the merger of Archimedian fluid statics with that tradition had been proposed before Galileo by Giambattista Benedetti. The originality of *De motu* lay in weaving the concept of impressed force into the same fabric. The buoyant effect of the medium is mathematically identical to the artificial lightness of an impressed force. The motion of wood in water is identical to the motion of a stone thrown upward, except that the artificial lightness of the wood remains constant whereas the artificial lightness of the stone decays.

Among other things, Galileo's concept of effective weight allowed him (as it had allowed others before him) to accept motion in a void. By the Aristotelian formula, velocity in a void would be infinite because resistance would be nil. The absurdity of an infinite velocity had been one of Aristotle's arguments against the possibility of a void. Obviously that absurdity evaporated in the reformulation of the role of resistance.

> For in a plenum the speed of motion of a body depends on the difference between its weight and the weight of the medium through which it moves. And likewise in a void [the speed of] its motion will depend on the difference between its own weight and that of the medium. But since the latter is zero, the difference between the weight of the body and the weight of the void will be the whole weight of the body. And therefore the speed of its motion [in the void] will depend on its own total weight. But in no plenum will it be able to move so quickly, since the excess of the weight of the body over the weight of the medium is less than the whole weight of the body.[30]

More interesting than the treatment of the void was the treatment of velocity. Like Aristotle, like the Scholastics, the Galileo of *De motu* believed that the maintenance of a constant velocity implies the action of a constant motive power. It is true that the other terms of the equation had been modified. When Galileo said that speed in a void depends on the total weight of a body, he meant its specific weight. As all pieces of pine, large and small, rise in water with the same force and velocity, so they all fall in the void with the same force and velocity. If lead and wood move with different velocities, and if the difference increases in a denser medium which subtracts a greater proportion of the specific weight of wood than of lead, all pieces of lead, whatever their size, fall with the same velocity through the same medium, and all pieces of wood do the same, though their velocity differs from that of lead. Behind the point he disputed with Aristotle, however, looms the larger fact that Galileo still shared the conviction that a constant velocity of motion implies a constant motor. *Omne quod movetur ab alio movetur.* With its effort to measure impressed force by weight and its concentration on vertical motion, *De motu* displayed in its starkest form a fundamental aspect of this dynamics. The dynamics of *De motu* was a direct transposition of statics. Velocity was set directly proportional to (specific) weight. Speed, Galileo asserted, cannot be separated from motion.

> For whoever asserts motion necessarily asserts speed; and slowness is nothing but lesser speed. Speed therefore proceeds from the same [cause] from which motion proceeds. And since motion proceeds from heaviness and lightness, speed or slowness must necessarily proceed from the same source. . . . For if the motion is downward, the heavier substance will move more swiftly than the lighter; and if the motion is upward, that which is lighter will move more swiftly.[31]

The primary thrust of *De motu*, the attempt to quantify the concept of impressed force, served further to confound statics and dynamics. By defining impressed force as lightness, he set velocity proportional to specific weight at every point in the trajectory of a rising or falling body. 'Since, then, a heavy falling body moves more slowly at the beginning, it follows that the body is less heavy at the beginning of its motion than in the middle or at the end.'[32] The state of a body at the top of its trajectory is identical to that of a suspended one. In each case, the impressed lightness exactly balances the body's weight, with the result that the speed is nil. As the impressed lightness decays, the weight increases and with it the speed. Thus the mathematical treatment of heavy and light implied a complete mechanics, not only statics, but dynamics as well.[33]

Within the conceptual framework of *De motu*, a quantitative dynamics was possible because the long tradition of statics, stretching back to the ancient world, to Archimedes and Aristotle, had prepared the way for Galileo's conception of static force. The very word 'force', deriving

from the Latin '*fortis*' (strong, powerful), came to Galileo from mechanics, that is, from the science of machines. 'Force' generally designated what was applied to one end of a lever to raise a 'weight' or 'resistance' at the other.[34] Among other things, the usage expressed the distinction of natural and forced; the lever and the other simple machines were devices to lessen the strain of lifting weights against their natural tendency. In his early treatise on simple machines, Galileo displayed a general conception of static force. Whatever the force applied might be, the strength of a man, say, or that of a beast, he found no problem in replacing it mentally with a weight hung over a pulley.[35] Weight arises from the tendency of all bodies to move toward the centre of the earth. As such, weight was not conceived to be a force. Since force was held to oppose and balance weight, however, weight could serve as the measure of force. In *De motu*, Galileo sought further to generalise by making weight the measure of dynamic action as well.

Behind the whole of *De Motu* stands the image of the balance. If fluid statics provided the rationale for the concept of artificial lightness, the understanding of fluid statics depended on the balance, which itself supplied another image to explain unnatural lightness. 'Upward motion is caused by the extruding action of a heavy medium,' Galileo stated. 'Just as, in the case of a balance, the lighter weight is forcibly moved upward by the heavier, so the moving body is forcibly pushed upward by the heavier medium.'[36] The balance provided the ultimate foundation for the measurement of force by weight. If two weights are in equilibrium on a balance, and an additional weight is added to one side, that side moves down, not in consequence of its whole weight, but only in consequence of the added amount. 'That is the same as if we were to say that the weight on this side moves down with a force measured by the amount by which the weight on the other side is less than it. And, for the same reason, the weight on the other side will move up with a force measured by the amount by which the weight on the first side is greater than it.'[37] In the case of natural motions, that is, the motions of heavy bodies through media that also are heavy, the weight of the body corresponds to one side of the balance and the weight of an equal volume of the medium to the other.

> And since the comparison of bodies in natural motion and weights on a balance is a very appropriate one, we shall demonstrate this parallelism throughout the whole ensuing discussion of natural motion. Surely this will contribute not a little to the understanding of the matter.[38]

To the scientist seeking to develop a mathematical mechanics at the beginning of the seventeenth century, the law of the lever represented the one secure quantitative relationship. Surely, as Galileo said, it would contribute not a little to understanding motion. When *De motu* attempted to transpose statics into dynamics, the statics it used derived from the simple machines. *De motu* took the law of the lever and attempted to expand it into a whole mechanics. If his mature system of mechanics was more subtle, the role played by the balance and lever was scarcely less central, and so it continued to be in mechanics through the rest of the century.

*

Sometime in the decade and a half following its composition, Galileo rejected the mechanics of *De motu*. No doubt his growing commitment to Copernican astronomy helped to dictate the rejection. Ironically, one thing that the quantified theory of impressed force could not explain was the phenomenon that the qualitative theory had been developed to explain, the phenomenon that had to be explained if the Copernican system were to be vindicated. Taken in its own terms, *De motu* had nothing to say about the horizontal motion of projectiles. It did discuss such projectiles briefly, but it did not face the issue of how an impressed force that is artificial lightness can cause any motion except one straight up. The quantification of impressed force was tied to its being lightness; if it were lightness, however, it was confined to explaining a class of motions so restricted as to be practically non-existent. *De motu* also mentioned that horizontal motion, like the rotation of a sphere at the centre of the universe, is neither natural nor forced, so that the smallest force can make a body move on a horizontal plane. In the suggestion, we can see Galileo's first step toward the concept of inertia. In *De motu*, however, it stood irreconcilably in conflict with the underlying premises on which the treatment of vertical motion rested. In Galileo's mature system of mechanics, expressed in his two master works, the *Dialogue Concerning the Two Chief World Systems* (1632) and the *Discourses and Demonstrations Concerning Two New Sciences* (1638), inertial motion, or uniform motion on a horizontal plane, did not replace *De motu's* concept of impressed force. Quite the contrary, it addressed itself to a range of phenomena that *De motu* had been powerless to handle adequately.

Instead of impressed force, uniform motion replaced the concept of rest as it was expounded in *De motu*. Perhaps I should rather say that Galileo expanded the brief suggestion that horizontal motion is like rest in its equilibrium of forces. He defined rest as infinite slowness, a special case of uniform motion. Within the context of Copernican astronomy the idea that the rotation of a sphere is neither natural nor forced took on added significance. Rotation on an axis involves no change of place; revolution in a closed orbit entails no rearrangement of positions. Rectilinear motion, in contrast, translates a body from one place to another, with the consequence either that the body was outside its proper place before its motion began or that it now moves away from its proper place. Circular motion, and circular motion alone, Galileo asserted, is compatible with an ordered universe.

> This being the motion that makes the moving body continually leave and continually arrive at the end, it alone can be essentially uniform. For acceleration occurs in a moving body when it is approaching the point toward which it has a tendency, and retardation occurs because of its reluctance to leave and go away from that point; and since in circular motion the moving body is continually going away from and approaching its natural terminus, the repulsion and the inclination are always of equal strengths in it. This equality gives rise to a speed which is neither retarded nor accelerated; that is, a uniformity of motion.[39]

In circular motion, the tendency toward the centre is exactly balanced by the repulsion from it. *De motu* had described rest as a similar state of equilibrium. Within Galileo's ordered, circular universe, all points equally removed from their relevant centres were equivalent. A body moving on the spherical surface through such points was effectively at rest; it could continue forever without introducing disorder into the cosmos. So overpoweringly obvious to the seventeenth-century mind was this conception of circular motion as dynamically equivalent to rest that it frustrated successful analysis of the mechanics of circular motion long after the nicely ordered cosmos that Galileo pictured had dissolved away.[40] Circular motion, which appeared so natural in the context of the Aristotelian world view as to be the symbol of perfection, became an enigma in the mechanical universe. Until its riddle was solved, a workable dynamics was impossible.

The ideal of a quantified, mathematical science of mechanics, the animating notion behind *De motu*, did not become less prominent in the reformulated mechanics of the *Dialogue* and the *Discourses*. The concept of uniform horizontal motion played a major role in its realisation. Without it, Galileo could not have demonstrated that the trajectory of a projectile is parabolic. The reformulation of his views on vertical motion played an even more important role, however, and one more directly concerned with the problem of force.

The conviction that uniform motion requires no cause bears some relation to the concept of uniformly accelerated motion, of course, although the relation is closer in our minds, with Newton's laws before us, than it was in Galileo's. The relation cannot be eliminated in Galileo's case, however. 'The stronger the cause, the stronger will be the effect,' he had asserted in the original version of *De motu*. 'Thus, a greater, that is, a swifter, motion will result from a greater weight, and a slower motion from a smaller weight.'[41] Compare the assertion with another, quoted above, made some ten years later in an essay on accelerated motion. Though the principle of motion in free fall remains constant, he declared, the velocity does not. 'It is necessary then to place the identity, or if you prefer the uniformity and simplicity, not in the velocity but in the increments of velocity, that is, in the acceleration.'[42] To us, the statement almost cries out with the assertion that force operates to change uniform motion so that a constant weight produces a uniform acceleration; although it did not cry out in the same way to Galileo, he probably could not have written the statement without the prior conviction that uniform motion requires no cause. Certainly it contradicts the earlier statement that a swifter motion requires a stronger cause. Equally his realization that a falling body cannot press on another falling with it, as it would if they were at rest, so that the weight of a falling body exerts itself entirely, as it were, in generating a uniformly accelerated motion, seems successfully to distinguish dynamics from statics and to state the relation of the two in acceptable form.[43] The appearance of both passages alters somewhat, however, when they are considered, against the background of *De motu*, as aspects of Galileo's correction of the internal contradictions in his early effort to frame a quantitative mechanics.

Even aside from the question of horizontal motion, the problems internal to *De motu* were acute. In constructing a mechanics that could describe free fall in quantitative terms, Galileo had at once accepted and amended Aristotle. When he set velocity directly proportional to weight, he reasserted the basic proposition of Aristotelian dynamics. If the proposition were granted, if he meant the assertion stemming from

it that a stronger cause produces a stronger effect, there appears to be no way of avoiding the conclusion that a heavier body falls more swiftly than a lighter one. Galileo was convinced, however, that all bodies of the same material, regardless of their size, fall with the same velocity. Two bodies of the same material, identical in size, obviously fall side by side with the same speed. According to Aristotle's mechanics, Galileo said, their speed will be doubled if, instead of falling side by side, they are joined together into one body twice as big. This is absurd; clearly the larger body will fall with the same velocity as its two parts. To meet the problem, *De motu* defined 'weight' in a peculiar way. Weight referred, explicitly, to specific gravity. Pieces of wood, he concluded, fall with one characteristic speed, pieces of lead with another, greater speed. At the same time, he held that all bodies are heavy because matter as such has a natural tendency toward the centre of the earth. Moreover, there is 'a single kind of matter in all bodies'; denser bodies merely enclose more particles of the same matter in equal spaces.[44] If that is the case, the argument which established that all pieces of lead fall with the same speed should have demonstrated as well that all bodies whatever fall with the same speed. The admission of that conclusion would have demolished the entire structure of *De motu*. On the one hand, it would have contradicted the principle that a stronger cause produces a stronger effect. On the other hand, it would have negated the explanation of acceleration in free fall, one of the central purposes of *De motu*. Impressed force was an artificial lightness, like the buoyant effect of water which alters the effective specific weight of a body; and the body's speed at every moment was set proportional to its effective specific weight. To conclude that all bodies fall with the same speed was to abolish acceleration in free fall, an unlikely foundation on which to base a mathematical dynamics.

In a word, the mechanics of *De motu* was shot full of irresolvable contradictions, and it is possible to see the mature analysis of uniformly accelerated motion, not primarily as a consequence of the concept of inertia, but rather as a reformulation of the imperfect *De motu*, a reformulation directed to the same ends the early treatise had pursued. When we think of uniformly accelerated motion in the context of inertia, we think immediately of a constant force constantly producing new increments of motion which are added to those produced before and conserved as inertial motion. Inevitably, such a view suggests Newton's second law, and when scientists came to view accelerated motion in these terms, Galileo's analysis of free fall was seen to furnish the prototype of the action of force. Galileo himself did not express his conception of accelerated motion in such a way. He spoke instead of the natural tendency of heavy bodies to move toward the centre of the earth with a uniformly accelerated motion.

> Every body constituted in a state of rest but naturally capable of motion will move when set at liberty only if it has a natural tendency toward some particular place; for if it were indifferent to all places it would remain at rest, having no more cause to move one way than another. Having such a tendency, it naturally follows that in its motion it will be continually accelerating. Beginning with the slowest motion, it will never acquire any degree of speed without first having passed through all the gradations of lesser speed . . . Now this acceleration of motion occurs only when the body in motion keeps going, and is attained only by its approaching its goal.[45]

In briefer form, 'a heavy body has an inherent tendency to move with a constantly and uniformly accelerated motion toward the common centre of gravity . . .'[46] The natural tendency toward the centre was identical to the natural tendency of *De motu*, although the motion it generated and the centre toward which it inclined had changed. In the Copernican world, the centre of the earth could not be the centre of the universe as *De motu* had implied, and Galileo duly insisted that there are multiple centres. The motion deriving from the tendency toward a centre was now uniformly accelerated. Acceleration was seen less as a dynamic effect than as a logical consequence of some implicit principle of continuity whereby a body at rest can acquire a given velocity only by passing through all the lesser degrees of velocity. Whereas there was one natural motion in *De motu*, there were now two, and Galileo was consistent in referring to them as two different motions. Thus his view of inertial motion contained a basic ambiguity. On the one hand, it was the motion that is neither natural nor forced, the motion involving neither a propensity nor a repugnance. On the other hand, it was something more: as a 'natural motion' it was a unique phenomenon with a cosmic purpose. The natural motions were necessary to an ordered universe built on Copernican lines. Circular motion could persevere forever without change, but could never generate itself; rectilinear motion toward (or away from) a centre was necessary to generate new motion (or to destroy it).

> [A] falling body starting from rest passes through all the infinite gradations of slowness; and . . . consequently in order to acquire a determinate degree of velocity it must first move in a straight line, descending by a short or long distance according as the velocity to be acquired is to be lesser or greater, and according as the plane upon which it descends is slightly or greatly inclined. . . . In the horizontal plane no velocity whatever would ever be naturally acquired, since the body in this position will never move. But motion in a horizontal line which is tilted neither up nor down is circular motion about the center; therefore circular motion is never acquired naturally without straight motion to precede it; but, being once acquired, it will continue perpetually with uniform velocity.[47]

God himself apparently chose the natural means of imparting orbital velocities to the planets. Galileo claimed to be following Plato in stating that there is a single point from which all the planets, in falling toward the sun, would attain precisely the right speeds at the levels of their orbits. If they were deflected into circular (horizontal) motion around the sun at that point, they would continue in it naturally forever.

Galileo insisted that a naturally accelerated body passes through every degree of velocity. Although the conception of accelerated motion implied no limit, his belief in an ordered universe did, and he spoke, for example, of the preternatural speed of a cannon ball which the resistance of the air destroys. In *De motu*, the existence of an upper limit to natural motion was implicit in the very structure of the treatise. What was more important, every degree of speed below that limit corresponded to an effective specific gravity. Since the gradations of specific gravity were infinite, so were the gradations of speed. Galileo's insistence that an accelerating body passes through every degree of velocity replaced the scale of weights in the earlier treatise.

In nothing is the continuity of the mature physics with *De motu* more evident than in Galileo's conviction that the natural tendency of heavy bodies to fall can provide the standard of measurement for a quantitative mechanics. Clearly the system of *De motu*, in which weight was the measure both of impressed force and of velocity, had to be revised. The major step forward in the *Discourses* was to replace the earlier statical mechanics with a true dynamics. Heavy bodies at rest, he had concluded, must be distinguished from heavy bodies in motion. The naturally accelerated motion of heavy bodies, however, could also be adapted to measuring other quantities in mechanics. Whereas velocity had been measured by weight, he now suggested that it be measured by sublimity, the vertical distance through which a body must fall in order naturally to attain that velocity. By this means a standard of velocity, which is constant everywhere, could be provided,

> since this velocity increases according to the same law in all parts of the world; thus for instance the speed acquired by a leaden ball of a pound weight starting from rest and falling vertically through the height of, say, a spear's length is the same in all places; it is therefore excellently adapted for representing the momentum acquired in the case of natural fall.[48]

In uniform motion, such a speed will carry a body through twice the distance of fall in the same period of time.

He sought as well to use the fact of naturally accelerated motion to establish a standard for measuring the force of percussion. The problem of percussion bothered Galileo, as it was to bother many others until Huygens restated it in more tractable terms and solved it. As Galileo first attacked the problem, he imagined a stake being driven into the ground. One blow of a hammer drives it a certain distance, and a dead weight considerably heavier than the hammer drives it an equal distance. Is the dead weight then a measure of that force of percussion? Galileo decided it is not, since a second blow of the hammer, identical to the first, will drive the stake the same distance again whereas more weight will have to be added to drive it further. Moreover, he found ambiguity in the role of the stake as a resistance. Suppose that the ground becomes harder with greater depth so that apparently equal blows drive the stake different distances. Can we say that the blows are in fact equal? Galileo decided they are not. The motion of the stake takes off some of a blow's force, and since the stake moves different distances, the blows cannot be equal. In searching for a standard by which to measure both resistance to a blow and the force of a blow, Galileo returned to the natural motion of heavy bodies. What force is constant? The force of a body which has fallen from a given height. What resistance is constant? The resistance of a heavy body to being raised. Galileo's final arrangement to examine the force of percussion consisted of a rope over a pulley with a weight on either end. The larger weight rests on a table. The other is dropped, and the force of its blow when it snaps the rope is transmitted over the pulley to the other weight. Because velocity can compensate weight, Galileo concluded,

and because the one body is at rest in the beginning, the ratio of weights is always smaller than the ratio of velocities. However small the falling body may be, it cannot fail to move the larger body at rest – a conclusion drawn from an analysis containing at least as much ambiguity as that which he sought to avoid.[49] Be that as it may, the natural tendency to descend had furnished the means to measure the force of percussion.

The fact that Galileo tried first to compare the force of percussion with a dead weight suggests that the system of *De motu* may have been less fully rejected than we have believed. By and large, however, the quantity which he referred to as *impeto* or *momento*, a quantity also connected with the natural tendency of heavy bodies, as the analysis of percussion shows, replaced weight as the measure of force. The quantity is similar to what is called 'momentum' today, but it differs in two respects. For one thing, Galileo did not possess a concept of mass distinct from that of weight. The second difference is far more important. As I indicated earlier, *impeto* (or *momento*) was usually employed with an active verb. Its appearance as the 'force of percussion' illustrates that it was central to the problem of force in Galileo's mature mechanics. Let me repeat that the word 'force' (or '*forza*' or '*vis*') is not the crucial consideration, but rather the conception employed whatever the word. If his own age saw a conception of force in Galileo's mechanics, it was one expressed by *impeto* and *momento*. Certainly it recalls the medieval doctrine of impetus and reminds us how profoundly that doctrine embodied our perception of the force of a moving body. Nevertheless, Galileo's use of *impeto* and *momento* was different from the medieval use. Like the impressed force of *De motu*, *impeto* was directed, not toward horizontal motion, but toward vertical motion by which it was measured. In this it revealed that its true forebear was less the concept of impetus than the concept of force applied to a simple machine to move a heavy body against its nature. Perhaps nothing is more revealing of seventeenth-century mechanics than the fact that the basic dynamic concept of the man who formulated the new conception of motion and defined uniformly accelerated motion referred, not to an external action changing a body's state of motion or rest, but to the capacity of a body in motion, its force, to raise itself again to the height from which it fell in acquiring that motion.

One of the most important propositions in Galileo's final system was the assertion that 'a heavy body falling from a height will, on reaching the ground, have acquired just as much *impeto* as was necessary to carry it to that height; as may be clearly seen in the case of a rather heavy pendulum which, when pulled aside fifty or sixty degrees from the vertical, will acquire precisely that speed and force which are sufficient to carry it to an equal elevation save only that small portion which it loses through friction on the air.'[50] As one consequence of the proposition, the path of descent was seen to be irrelevant to the *impeto* acquired (in the ideal case, of course, in which friction is excluded). Vertical displacement alone governs the speed acquired, so that bodies descending by different inclined planes between the same pair of horizontal planes acquire the same speed. Initially, Galileo introduced the proposition as an assumption; the second edition of the *Discourses* included a passage, written by Viviani under Galileo's instruction, which attempted to demonstrate it from dynamic considerations.[51] Why should it have occurred to him in the first place? Because, I suggest, it embodied a dynamic equivalent of the commensurability of impressed force and weight in *De motu*. A body is at rest, the early treatise had asserted, when the impressed force striving to raise it equals the weight urging it down. The *impeto* of a moving body, the *Discourses* held, is just enough to overcome the natural inclination to descend while it raises the body to its original position. Because circular motion is equivalent to rest, any place on the horizontal plane around the gravitational centre is equivalent to its original position. Thus *De motu's* measurement of impressed force by weight was replaced in the mechanics of the *Discourses* by a measurement of *impeto* which was also based on the natural inclination of heavy bodies to descend.

The measurement of force as *impeto* is inconsistent with the concept of force we see implicit in the analysis of uniformly accelerated motion. To express it in our terms, force as *impeto* would be equal, not to the rate of change of momentum, but to the momentum itself or to the total change of momentum, Δmv. Both usages, despite their inconsistency, appear side by side in Galileo. The first is implicit in his concept of the percussive force of a body, and the second appears with the assumption that this force must be equal to the force which can generate it. When a ball is rolled up an inclined plane, 'a given movable body thrown with a given force [*forza*] moves farther according as the slope is less.' In discussing the range of projectiles fired at different elevations, Galileo demonstrated that 'less momentum' [the Latin is "*impetus*"] is required 'to send a projectile from the terminal point *d*

along the parabola *bd* [with an elevation of 45°] than along any other parabola having an elevation greater or less . . .' He spoke also of the difference of '*impeti*' and '*forze*' when projectiles are fired over the same range at different elevations.[52] Force in this context is measurable only as Δmv.

The notion of *impeto* or *momento*, so prominent in Galileo, illustrates the continued domination of mechanics by the law of the lever, even in the case of the man who first effectively distinguished statics and dynamics. In his *Discourse on Bodies in Water*, he justified the definition of *momento* by reference to the balance.

> As for example, two weights equall in absolute Gravity, being put into a Ballance of equall Arms, they stand in Equilibrium, neither one going down, nor the other up: because the equality of the Distances of both, from the Centre on which the Ballance is supported, and about which it moves, causeth that those weights, the said Ballance moving, shall in the same Time move equall Spaces, that is, shall move with equall Velocity, so that there is no reason for which this Weight should descend more than that, or that more than this; and therefore they make an Equilibrium, and their Moments continue of semblable and equall Vertue.[53]

If a lesser weight is to raise a greater, it must be placed further from the fulcrum so that it moves more swiftly.

> Generally then we say that the moment of the lighter body equals the moment of the heavier when the velocity of the smaller stands in the same proportion to that of the larger as the weight of the larger to that of the smaller. . . .[54]

The passages above indicate how our concepts of momentum and moment both derived from the single term *momento* as Galileo and many others used it.

In Galileo's exposition, the concept of *momento* continually strained to free itself from bondage to the balance and lever. In his *Discourse on Bodies in Water*, he employed the principle to demonstrate that a beam of wood can float in a volume of water one hundredth as great. The beam, of course, is in a narrow tank surrounded by a curtain of water, so that a small vertical motion by the beam causes a large fluctuation in the narrow sheets of water along its sides. The demonstration might seem paradoxical, he concluded. 'But he that shall but comprehend of what Importance Velocity of Motion is, and how it exactly compensates the defect and want of Gravity, will cease to wonder . . .'[55] When he treated the force of percussion, he employed the same principle, now wholly separated from any constraining mechanism. In his early treatise *On Mechanics*, he suggested that the 'resistance to being moved' of the hammer bears the same proportion to the 'resistance to being moved' of the object it strikes as the distance the force would drive the hammer if it did not strike to the distance the object is driven.[56] By the time of his final deliberations on percussion, Galileo no longer spoke of a general resistance to being moved, but the tentative conclusion at which he arrived repeated the law of the lever. In the arrangement of two bodies over a pulley, he compared the ratio of the bodies to the ratio of their speeds, and decided that a body no matter how small will move another since the velocity of the other is initially zero.

The balance and the lever contained a basic ambiguity which further confused the conception of force. The problem was present already in his early *Mechanics*, where he treated the lever on the principle that

> whatever is gained in force is lost in speed. For the force *C* elevating the lever and transferring it to *AJ*, the weight is moved through the interval *BH*, which is as much less than the space *CJ* traversed by the force as the distance *AB* is less than the distance *AC*; that is, as much as the force is less than the weight.[57]

What is the significant factor, the speed or the displacement? In the passage above, and in many others, Galileo used the two interchangeably, as of course they can be used interchangeably in the case of simple machines. The problem lay in the possibility of forgetting that the interchangeability is valid only when the two motions being compared are virtual motions constrained to a simple machine. On the whole, Galileo kept the condition in mind, but the possibility of forgetting was built into the very structure of his crucial proposition, that a body in falling acquires *impeto* sufficient to carry it back to its original height. Here he was dealing with real, accelerated motion, not with virtual motion, but he yoked displacement and velocity together in a way all too likely to suggest their interchangeability. On at least one occasion, Galileo demonstrated how readily the two might be confused. In the so-called sixth day of the *Discourses*, he applied the law of the inclined plane (which he derived directly from the lever) to the question of percussion. A body of ten pounds descending vertically balances one of a hundred pounds on an inclined plane the length of which is ten times its elevation. Therefore, he continued,

> drop a body of ten pounds through any vertical distance; the *impeto* it acquires, when applied to a body of one hundred pounds, will drive it an equal distance up the inclined plane, which involves a vertical rise equal to one tenth the length of the inclined plane. It was concluded above that a force able to drive a body up an inclined plane is sufficient to drive it vertically a distance equal to the elevation of the inclined plane, in this case a tenth part of the distance traversed on the incline, which distance on the incline is equal to the fall of the first body of ten pounds. Thus it is manifest that the vertical fall of a body of ten pounds is sufficient to raise a body of one hundred pounds vertically, but only through a space equal to one tenth of its descent.[58]

Galileo applied the conditions of equilibrium on an inclined plane to the accelerated motions of two independent bodies and tried to connect the two by the concept of *impeto*. What is the measure of force in the case above? Is it the *impeto* (in our terms, mv) or is it the product of weight times vertical displacement, what we call work, and what Leibniz used to measure *vis viva* (mv^2). The two are not equivalent to each other. From the ambiguity of the lever emerged the controversy over *vis viva*. To us, with over two centuries to digest both Newton's achievement and Leibniz's, the ambiguity seems obvious enough. To the age that created modern mechanics, however, the problem of force was so complicated that the ambiguity only began to be manifest, much less to dissolve, in the second half of the century, a generation after Galileo's death.

Of course, a third use of force, incompatible both with mv and with mv^2 was also present in the analogy of the lever and in the examples above. This was the idea of static force, identical to that employed in *De motu*. Consider a passage from the 'Sixth Day' of the *Discourses*. Since a body in falling acquires enough *impeto* (mv) to raise it to its original height, a certain amount of *forza* (mv^2) is necessary to raise the body a given vertical height whatever the plane by which it rises. To pull it up different planes, however, a smaller *forza* (F) is needed as the inclination is less.[59] In discussing percussion, he asserted that the *forza* which can raise a weight is equal to the *forza* with which it presses down, and with that quantity he tried to measure the force of percussion.[60] Perhaps nothing expresses the ambiguity so well as the passage by Viviani, interpolated at Galileo's request in the second edition of the *Discourses* to justify the assertion that a given change of *impeto* corresponds to a given vertical displacement, whatever the path followed