International Association of Geodesy Symposia

Ivan I. Mueller, Series Editor

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Symposium 101: Global and Regional Geodynamics
Symposium 102: Global Positioning System: An Overview
Symposium 103: Gravity, Gradiometry, and Gravimetry
Symposium 104: Sea Surface Topography and the Geoid
Symposium 105: Earth Rotation and Coordinate Reference Frames

Global Positioning System: An Overview

Symposium No. 102

Edinburgh, Scotland, August 7–8, 1989

Convened and Edited by
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Foreword

A General Meeting of the IAG was held in Edinburgh, Scotland, to commemorate its 125th Anniversary. The Edinburgh meeting, which attracted 360 scientific delegates and 80 accompanying persons from 44 countries, was hosted jointly by the Royal Society, the Royal Society of Edinburgh and the University of Edinburgh.

The scientific part of the program, which was held in the Appleton Tower of the University, included the following five symposia:

Symposium 101	Global and Regional Geodynamics
Symposium 102	GPS and Other Radio Tracking Systems
Symposium 103	Gravity, Gradiometry and Gravimetry
Symposium 104	Sea Surface Topography, the Geoid and Vertical Datums
Symposium 105	Earth Rotation and Coordinate Reference Frames

All together there were 90 oral and 160 poster presentations. The program was arranged to prevent any overlapping of oral presentations, and thus enabled delegates to participate in all the sessions.

The 125th Anniversary Ceremony took place on August 7, 1989, in the noble surroundings of the McEwan Hall where, 53 years earlier, Vening-Meinesz gave one of the two Union Lectures at the 6th General Assembly of the IUGG. The Ceremony commenced with welcome speeches by the British hosts. An interlude of traditional Scottish singing and dancing was followed by the Presidential Address given by Professor Ivan Mueller, on 125 years of international cooperation in geodesy. The Ceremony continued with greetings from representatives of sister societies, and was concluded by the presentation of the Levallois Medal to Professor Arne Bjerhammar. The 125th Anniversary was also commemorated by an exhibition entitled *The Shape of the Earth*, which was mounted in the Royal Museum of Scotland. An abbreviated version of the President's speech and the list of all participants are included as appendices in this volume.

A social program enabled delegates to experience some of the hospitality and culture of both Edinburgh and Scotland, as well as provided an opportunity to explore the beautiful City of Edinburgh and the surrounding countryside.

A Scottish Ceilidh on the last night concluded a pleasant week, which was not only scientifically stimulating, but also gave delegates and accompanying persons an opportunity to renew *auld acquaintances* and make new ones.

The International Association of Geodesy and the UK Organizing Committee express their appreciation to the local organizers of the General Meeting, especially to Dr. Roger G. Hipkin and Mr. Wm. H. Rutherford, for their tireless efforts in running the meeting to its successful conclusion.

Commencing with these symposia the proceedings of IAG-organized scientific meetings will be published by Springer Verlag Inc., New York from author-produced camera-ready manuscripts. Although these manuscripts are reviewed and edited by IAG, their contents are the sole responsibility of the authors, and they do not reflect official IAG opinion, policy or approval.

V. Ashkenazi
A. H. Dodson
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Preface

Space geodesy has evolved in the last twenty years into one of the most exciting disciplines in the earth sciences. In particular, the last five years have seen unprecedented growth in this field with no end in sight. This development is due to a large extent to the versatility of applications provided by the radio interferometric technique called GPS. It is fair to say that the Global Positioning System has already revolutionized or will soon revolutionize almost all conceivable positioning applications (precise or otherwise, large or small scale, scientific or commercial) on land, in the air, in space, and at sea. And the GPS constellation is only beginning to approach its full operational phase with the launching of the first five Block II satellites by the end of 1989.

When we were asked to convene Symposium 102 on *The Global Positioning System and Other Radio Tracking Systems* at the IAG General Meeting in Edinburgh, we thought it appropriate to present a broad review of GPS geodesy, achievements to date and prospects for the future. A natural way to accomplish this was through the active participation of the IAG special study groups and subcommissions whose charters were related to the scope of the symposium, in particular SSG's 1.104—Static and Geodynamic Positioning with GPS—Y. Bock, president; 1.105—GPS Kinematic Positioning Methods and Applications —G. Lachapelle, president; 2.54—Determination of Orbits to Cm-Accuracy—O. Colombo, president; 4.117—Optimization of Modern Positioning Techniques—D. Delikaraoglou, president; and the CSTG Global Positioning System Subcommission—G. Mader, president. The presidents of these groups were asked to serve on the organizing committee and were invited to deliver keynote presentations and to invite other speakers. We decided on this approach because of the limited time (a day and a half) allocated to a symposium whose scope covers such an active research area, and the large number of contributed papers that were anticipated. We felt that review papers of a half-hour length would provide the participants with a good overview of the field at a reasonable, nonrushed pace, with current research activity covered in detail at the poster boards. The interactions at the poster boards between the authors and participants have been found to be more conducive to fruitful exchange of ideas than a flurry of ten-minute oral presentations that leave all participants exhausted.

Symposium 102 was held on August 7–8, 1989, in Edinburgh, Scotland, as part of the International Association of Geodesy's 125th Anniversary General Meeting. The IAG call for papers produced 63 abstracts, which were all accepted for the poster sessions. Of these, 43 were eventually presented. The structure of the Symposium was a series of nine keynote papers presented in plenary session supported by the poster presentations. The posters were grouped to correspond with the keynote papers so that the plenary and poster sessions could be linked. The Symposium had the advantage of having all the posters displayed before, during and after the plenary sessions with the formal poster presentations after their plenary lead papers. Reducing the number of oral presentations in the Symposium enabled discussion periods to be meaningful. The Symposium attracted considerable attention with much interest in the poster presentations.

In this volume the papers are grouped into sections according to the relevant IAG special study groups and subcommission. In each section the first papers are the keynote presentations. They are intended as a comprehensive review of the current status of GPS geodesy with a look towards the future, from the perspective of the presidents of the GPS-related IAG study groups and subcommissions, and their colleagues.

The rest of the sections contain a collection of 29 papers based on the poster presentations. They provide an overview of current research activities in GPS geodesy and thereby complement the keynote presentations.

We would like to thank all the authors for their contributions to the realization of this volume and to the IAG Executive Committee for giving us the opportunity to organize Symposium 102 and prepare this document.

Yehuda Bock　　　　Norman Leppard

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Campaigns

Appendix 1 — Presidential Address

RELATIVE STATIC POSITIONING WITH THE GLOBAL POSITIONING SYSTEM: BASIC TECHNICAL CONSIDERATIONS

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1. INTRODUCTION

We give an overview of some of the important everyday-problems of using the Global Positioning System (GPS) for relative positioning in networks of small to medium size (let us say from 5×5×2 km to 100×100×2 km). We assume that the surveys are performed with single- and dual-frequency receivers. We allow for different receiver types and we assume that at least some of the dual frequency instruments are of C/A-code type (recovering the L_2-phase using some squaring technique). The questions we address are the following: (1) What linear combination should be used for a special problem type (e.g., for pre-processing, ambiguity resolution, final solutions)? (2) How are mathematical correlations handled correctly in the general case; are we ready to process upcoming events with 50-100 simultaneously operating receivers in a correct way? (3) What is the optimum way of using "basic" atmosphere information (surface meteorological data, use of dual frequency data for single frequency receivers working in the same area)? (4) What is the state of the art of combining receivers of different type? (5) If we are interested in moderate accuracies only: how long do we have to stay on a survey point to get a satisfactory answer. Or, more generally, how to perform a kinematic survey using the methods of static positioning?

When we browse through the proceedings of conferences on satellite positioning (e.g., Rockville 1985, Austin 1986) and the three major journals (*Journal of Geophysical Research*, *Bulletin Géodésique*, *Manuscripta Geodaetica*) for papers on the geodetic use of the Global Positioning System (GPS) we find that these may be grouped as follows:

(1) Accuracy demonstration of relative static positioning
(2) Orbit determination/satellite clock modeling
(3) Papers dealing with technical aspects:
  - Mathematical correlations between simultaneous GPS observations
  - Special linear combinations of the L_1 and L_2 carrier phases
  - Ambiguity resolution
(4) Influence of unmodeled biases on geodetic results (e.g., orbits, atmosphere)
(5) Kinematic surveys

Let us have a closer look at some of these contributions:

(i) The papers of group (1) tell us that there are remarkable improvements since 1982: From a few parts per million and a few millimeters on very short baselines (Bock *et al.*, 1984; *Goad et al.*, 1984) the accuracy in the horizontal position came down to a few parts per 10 millions (e.g., Beutler *et al.*, 1987) to a few parts in 100 millions (e.g., Lichten and Border, 1987). The step from 1 ppm to 0.1 ppm was possible due to the availability of dual frequency receivers and orbit improvement techniques, the step from 0.1 to 0.01 ppm due to the consequent application of the fiducial point concept (Kroger *et al.*, 1985) and refined modeling techniques (Lichten and Border, 1987) to determine the orbits of the GPS satellites. It also became obvious that high relative accuracies are more easily obtained on long baselines (the ideal situation would be an accuracy not depending on the baseline length). We found (Beutler *et al.*, 1989]) that at present the baseline accuracy obtainable with GPS is quite well reflected by the "law"

$$\frac{db}{b} = \sqrt{\frac{1}{2b}} \quad \text{mm/km} \tag{1.1}$$

where: b is the baseline length in km
db is the error in (one of) the baseline components (in mm).

(2) Orbit determination using the observations of a regional fiducial tracking network is an absolute prerequisite for obtaining GPS results of the order of 0.01 ppm in a regional or continental network. A very careful analysis of this problem is given by Lichten and Border (1987), a review of the same topic is included in these proceedings by Lichten. It seems worthwhile to mention that this problem may become significantly easier to deal with when high precision code measurements become available at the fiducial sites (Jet Propulsion Laboratory's ROGUE receivers promise code accuracies of 5 cm (see, e.g., Blewitt *et al.*, 1988): The time consuming preprocessing and ambiguity resolution steps will be superfluous. This is an important aspect for routine procedures.

Kinematic surveys which are outside the scope of the present article will be dealt with in detail in these proceedings by Lachapelle and Schwarz. It is interesting however that this topic received so much attention. Obviously the goal of kinematic surveys, to obtain a moderate point accuracy (of the order of 1–3 cm) in a short time, is most attractive to many people. For us this was the reason to include section 6, where we try to solve the same problem, using the methods of static positioning only. This problem has promising solutions if, and only if, we manage to resolve the initial phase ambiguities using a short observation time span.

Let us have a quick look at the remaining sections:

In section 2 we summarize the essential knowledge concerning linear combinations of dual frequency (code and) phase observations.

In section 3 we review the state of the art of modeling mathematical correlations between simultaneously observed phase (or code) observations. For the sake of completeness we include the background necessary to handle this problem in the most general case when single- and dual-band receivers are combined in the same adjustment process (there are users doing this on a routine basis).

In section 4 we look into the problem of atmospheric refraction. We start with a brief literature survey, then we put the emphasis on elementary, but as we think, important practical aspects.

Experienced GPS users may have noticed that in small networks baseline- or network-repeatability is usually better when actual meteorological measurements are discarded and not used. We show that there are better ways of using these measurements than to introduce them into the well known Saastamoinen or Hopfield models.

We will also present practical ways to estimate models of ionospheric refraction using the dual band phase (or code) observations of one or more receiver(s). These models, when used for single or dual frequency surveys in the neighborhood, will in most cases reduce ionospheric biases well below the residual level of the observations..

In section 5 we present the current knowledge of combining receivers and antennae from different manufacturers. This problem will become more and more important because there undoubtedly will be many more products on the market. From the economical point of view it is absolutely mandatory to combine observations from all these receivers.

2. FORMING LINEAR COMBINATIONS

Linear combinations of the basic GPS observables

- phases ϕ_1 and ϕ_2
- (P- or C/A-) codes P_1 and P_2

on the two carriers L_1 and L_2 are used for partial or complete elimination of biases and nuisance parameters in the functional model. Such linear combinations can be formed in several ways

- using observations on the same carrier and of the same type but stemming from different receivers, satellites, and epochs (single-, double-, triple-differences)

- using observations of the same type on the two carriers

- using observations of different type (code and phase).

The main purpose of forming single-, double-, and triple-differences is the elimination of satellite clock errors, receiver clock errors (largest part) and initial phase ambiguities, respectively. If the mathematical correlations introduced by such differencing techniques are handled rigorously this procedure leads to the same results as processing undifferenced observations introducing the respective biases as additional unknown parameters (see section 3).

We could even speak of linear combinations of the primary unknowns (the station coordinates), namely the "differential application of GPS." This differencing is usually done implicitly by keeping the coordinates of one station fixed yielding a great reduction of the influences of orbit errors and (to a certain extent) atmospheric refraction.

The main topic of this section is the linear combination of observations on different carriers. The main reasons to form these combinations are the following:

- Elimination of ionospheric refraction to avoid biased solutions.

- Ionospheric refraction and unmodeled orbit errors may make ambiguity resolution on longer baselines difficult or even impossible when using L_1 and/or L_2 observations.

In the following, we describe linear combinations that either lead to solutions unbiased by the ionosphere or help ambiguity resolution on medium range baselines.

Let us start with the basic single-difference phase observation equation involving two stations i and k (the phase observations ϕ_f, f=1,2 have been replaced by their metric representation $\ell_f = \phi_f \cdot \lambda_f$, λ_f being the wavelength of the carrier L_f, f=1,2):

$$\Delta\ell_f + v_f = \Delta\rho + (c-\dot{\rho}_i)\,\Delta t + \Delta\delta_f^{ION} + \Delta\delta^{TROP} + \lambda_f \cdot N_f \tag{2.1}$$

where

Δ	:	single difference operator (stations i and k)
v_f	:	residual of observation ℓ_f
N_f	:	integer ambiguity
ρ	:	range to the satellite
$\dot{\rho}_i$	:	range rate with respect to station i
Δt	:	synchronization error between receivers
δ_f^{ION}	:	ionospheric refraction
δ^{TROP}	:	tropospheric refraction
c	:	speed of light

Note that by forming the double-difference observation equation (difference of eqns. of type (2.1) for two different satellites j, m) the receiver clock synchronization error still remains in the equation with a coefficient of :

$$\dot{\rho}_i^j - \dot{\rho}_i^m$$

That means that the synchronization error has to be known to about 1 microsecond, which is easily obtainable (even in real-time) using the code measurements. The frequency-dependent ionospheric refraction (meters) is given by

$$\Delta\delta_f^{ION} = -\frac{41 \cdot (E_i - E_k)}{v_f^2} = \frac{K}{v_f^2}; \qquad f = 1, 2 \tag{2.2}$$

where: E_i and E_k are the total electron contents of the ionosphere in a cylinder of 1 m^2 cross section along the lines of sight from the two stations i, k to the satellite,

v_f is the frequency of the carrier f.

The general linear combination of two single-differences (same station, same satellite) observed in L_1 and L_2 may be written as

$$\Delta\ell_j = \chi_{j,1} \cdot \Delta\ell_1 + \chi_{j,2} \cdot \Delta\ell_2 \tag{2.3}$$

Using the error propagation law (assuming no correlation between $\Delta\ell_1$ and $\Delta\ell_2$) we may compute the corresponding noise

$$m_{\Delta\ell_j} = \sqrt{\chi_{j,1}^2 \cdot m_{\Delta\ell_1}^2 + \chi_{j,2}^2 \cdot m_{\Delta\ell_2}^2} \tag{2.4}$$

Or, if we postulate identical standard deviations in the phases (measured in cycles)

$$m_{\Delta\phi_1} = m_{\Delta\phi_2} = m_{\Delta\phi}$$

we obtain

$$m_{\Delta\ell_1} = \lambda_1 \cdot m_{\Delta\phi}$$

$$m_{\Delta\ell_2} = \lambda_2 \cdot m_{\Delta\phi} = \frac{\lambda_2}{\lambda_1} \cdot m_{\Delta\ell_1} = \frac{\nu_1}{\nu_2} \cdot m_{\Delta\ell_1} \tag{2.5}$$

and finally:

$$m_{\Delta\ell_j} = \sqrt{\chi_{j,1}^2 + \frac{\nu_1^2}{\nu_2^2} \chi_{j,2}^2} \cdot m_{\Delta\ell_1} \tag{2.6}$$

Starting from the observations $\Delta\ell_1$ and $\Delta\ell_2$ we present in Table 2.1 a summary of the coefficients of the five most important linear combinations, together with the influence of frequency-independent biases (e.g., troposphere, orbits), frequency-dependent biases (ionosphere), and error propagation of the observation noise, all expressed in "length units" as well as cycles, relative to an assumed size of 1 in $\Delta\ell_1$.

LC	λ	LC-Factors		Tropospheric/Orbit Biases		Ionospheric Biases		Observation Noise	
	[m]	$x_{\ell,1}$	$x_{\ell,2}$	[length]	[cycles]	[length]	[cycles]	[length]	[cycles]
L_1	c/ν_1 0.19	1	0	1	1	1	1	1	1
L_2	c/ν_2 0.24	0	1	1	$\frac{\nu_2}{\nu_1}$ 0.78	$\frac{\nu_1^2}{\nu_2^2}$ 1.6	$\frac{\nu_1}{\nu_2}$ 1.3	$\frac{\nu_1}{\nu_2}$ 1.3	1
L_3	0	$\frac{\nu_1^2}{\nu_1^2-\nu_2^2}$ 2.5	$\frac{-\nu_2^2}{\nu_1^2-\nu_2^2}$ -1.5	1	-	0	-	$\frac{\nu_1\sqrt{\nu_1^2+\nu_2^2}}{\nu_1^2-\nu_2^2}$ 3.1	-
L_4	∞	1	-1	0	-	$-\frac{\nu_1^2-\nu_2^2}{\nu_2^2}$ -0.6	-	$\sqrt{1+\frac{\nu_1^2}{\nu_2^2}}$ 1.6	-
L_5	$\frac{c}{\nu_1-\nu_2}$ 0.86	$\frac{\nu_1}{\nu_1-\nu_2}$ 4.5	$\frac{-\nu_2}{\nu_1-\nu_2}$ -3.5	1	$\frac{\nu_1-\nu_2}{\nu_1}$ 0.22	$-\frac{\nu_1}{\nu_2}$ -1.3	$\frac{\nu_2-\nu_1}{\nu_2}$ -0.28	$\frac{\nu_1}{\nu_1-\nu_2}\sqrt{2}$ 6.4	$\sqrt{2}$ 1.4
L_5'	$\frac{c}{\nu_1-2\nu_2}$ -0.34	$\frac{\nu_1}{\nu_1-2\nu_2}$ -1.8	$\frac{-2\nu_2}{\nu_1-2\nu_2}$ 2.8	1	$\frac{\nu_1-2\nu_2}{\nu_1}$ -0.56	$\frac{(\nu_2-2\nu_1)\nu_1}{(\nu_1-2\nu_2)\nu_2}$ 2.8	$\frac{\nu_2-2\nu_1}{\nu_2}$ -1.6	$\frac{\nu_1}{\nu_1-2\nu_2}\sqrt{5}$ -4.0	$\sqrt{5}$ 2.2

Table 2.1
Summary of the linear combinations (factors to apply to metric observations ℓ_1 and ℓ_2)

Comments on Table 2.1:

(1) The linear combination L_3 does not contain any ionospheric refraction, yielding therefore solutions unbiased by the ionosphere. However, the corresponding linear combination of the integer ambiguities will be a real-valued expression. The noise level of L_3 is about 3 times larger than the one of L_1 The latter fact may be a source of concern, especially on short baselines. Bock *et al.* (1986) and Schaffrin and Bock (1988) discuss this topic in some detail. They state that the "obvious" combination of L_1 and L_2 (eliminating ionospheric refraction) is not always the best choice because "... any source of noise which is dispersive as, e.g., multipath, will be amplified in the ionosphere free linear combination ... ." This is certainly true for small networks where the differential influence of the ionosphere may be very well accounted for by a simple model. Another possibility, also pointed out by Bock *et al.* (1986), would be to introduce into the processing, instead of the L_3 linear combination, the original L_1 and L_2 equations plus a weighted equation for the ionospheric contribution. Depending on the adopted weight the equation system may be varied between complete neglect and a rigorous elimination of the ionosphere.

(2) The linear combination L_4 contains the contribution of the ionosphere plus a real-valued linear combination of the ambiguities in L_1 and L_2 only. It is unaffected by any other influence like orbits, receiver clocks, troposphere. It may be used, e.g., for studies of ionospheric behavior.

(3) The linear combination L_5 is formed by differencing of the L_1 and L_2 phases (expressed in cycles). Its wavelength is about 86 cm ("wide-lane"), and its corresponding ambiguity $N_5 = N_1 – N_2$ keeps its integer nature. It can be used to determine the difference of the basic ambiguities $N_1–N_2$ on medium range baselines. In a further step we can introduce the known $N_1–N_2$ into the observation equations for L_3 to resolve the remaining N_1 ambiguity.

(4) If we want to process dual frequency observations from instruments which measured L_2 in units of half-cycles, we had two possibilities to form a "wide-lane type" of linear combination:

- We just form the L_5 linear combination of the observations ℓ_1 and ℓ_2 ($\ell_1= \phi_1 \cdot \lambda_1$, $\ell_2= \phi_2 \cdot \lambda_2/2$). The resulting linear combination of the ambiguities may be written as

$$(N_1 - \frac{1}{2}N_2)\cdot\lambda_5=(2N_1 - N_2)\cdot(\lambda_5/2)=N_5\cdot(\lambda_5/2)$$

which shows that we have to look for an integer multiple of $(\lambda_5/2)$=43 cm.

- We use another linear combination (L'_5 of Table 2.1), i.e., the difference of the original phase measurements (units of cycles, 1 cycle in L_1 corresponding to ≈ 19 cm) with a resulting wavelength of about 34 cm (Allison, 1988).

The latter linear combination however has distinct disadvantages for the resolution of the ambiguities: The influences of wavelength-independent error sources is (expressed in units of the ambiguities) about 1.3, the influence of ionospheric refraction about 2.8 times larger than in the former combination.

After having successfully resolved the ambiguities either of the linear combination L_5 (in multiples of $\lambda_5/2$=43 cm) or of the linear combination L'_5 (in multiples of λ'_5=34 cm), we may now use either the ionosphere-free linear combination or the original carriers L_1

and L_2 to solve for the N_1 ambiguities of the L_1 carrier. It is important that for this purpose we are looking for multiples of λ_1 and not, as one might think, for multiples of $\lambda_1/2$.

Linear Combinations of Phase and Code in L_1 and L_2: There were two remarkable papers about this subject at the 1985 Rockville Symposium (Melbourne, 1985; Wübbena, 1985). Both authors independently came up with the same idea: to use a special linear combination w of the two code measurements P_1 and P_2 (on L_1 and L_2) and the corresponding phases ℓ_1 and ℓ_2 to resolve the difference of the L_1 and the L_2 initial carrier phase ambiguities (N_1 - N_2). For a specific epoch they put

$$w = \chi_1 \cdot \ell_1 + \chi_2 \cdot \ell_2 + \chi_3 \cdot P_1 + \chi_4 \cdot P_2 \qquad (2.7)$$

where

P_1, P_2, ℓ_1, ℓ_2 are expressed in meters

$\chi_1 = \nu_1/(\nu_1-\nu_2), \quad \chi_2 = -\nu_2/(\nu_1-\nu_2)$

$\chi_3 = -\nu_1/(\nu_1+\nu_2), \quad \chi_4 = -\nu_2/(\nu_1+\nu_2)$

ν_1, ν_2 are the frequencies of the L_1, L_2 carriers

Ionosphere, geometry, satellite- and receiver-clocks cancel out completely when forming the right hand side of eqn. (2.7). What remains is a term proportional to the difference of the initial carrier phase ambiguities N_1-N_2. By collecting all eqns. (2.7) for all epochs (for a special satellite) it is therefore possible to give an estimate for N_1-N_2 unbiased by orbits, station coordinates, atmospheric refraction! The approach has proved to be very successful when processing TI-4100 observations; it will be even more powerful when in the future antennae less affected by multipath will be used.

3. MATHEMATICAL CORRELATION

Forming differences between (quasi-) simultaneous GPS observations is an essential, if not the most important, aspect of all GPS processing techniques: When forming the difference between observations of the same satellite made by two receivers (usually called single difference), the satellite clock error (and the orbit error) are greatly reduced. When forming the difference between the observations of two satellites recorded by the same receiver (let us call this an inter-satellite difference) the receiver clock error is greatly reduced. Therefore the functional model of the so-called double difference (inter-satellite difference of two single differences made by the same receivers) is much simpler than that of the original phase observations: satellite clocks usually do not have to be modeled (compare however section 5), receiver clocks have to be modeled only on the 1 microsecond level, which may be done easily using C/A- or P-code. As usual such advantages have their price. In this case the price is mathematical correlation between $(m-1)\cdot(n-1)$ linearly independent double differences formed from $m\cdot n$ uncorrelated phase observations (m is the number of stations, n the number of satellites at the observation time considered).

Correlations Within Single Baselines

Correlations are easily handled when processing data from a single baseline It is then possible to decorrelate the double difference observations of one epoch using, e.g., a

Gram-Schmidt scheme (Remondi, 1984) or to compute the weight matrices of all simultaneous double differences as the inverse of the corresponding variance-covariance matrix (Beutler *et al.*, 1984). These weight matrices may be computed once and for all if the between-satellite differences are always formed using the same pattern (e.g., satellite 1–2, 2–3, ..., or 1–2, 1–3, ...), because then the weight matrix is uniquely a function of the number of the satellites observed.

Correlations Within a Network

The problem is much more complex if data from more than two receivers are processed. Several algorithms were discussed for that purpose. It is important to state that the result will be identical in all cases:

(1) It is attractive to look at the problem from the physical point of view (Goad, 1985). The reason for forming differences are satellite- and receiver-clocks. An equivalent approach to differencing therefore is to use undifferenced phases (which are uncorrelated) and to introduce per epoch one clock parameter for each satellite and one for each station (but one). These epoch-specific parameters have to be pre-eliminated after each epoch, otherwise the number of parameters in the final adjustment would be far too large.

(2) It is also possible to keep the double difference as basic observable and, as in the single difference case, to compute the weight matrix pertaining to one epoch as the inverse of the corresponding variance-covariance matrix (Beutler *et al.*, 1986; Bock *et al.*, 1986). Very efficient algorithms could be developed when (at a specific epoch) the observation scheme is identical for each receiver. The analysis (and the algorithms) are becoming more and more tedious when more and more data outages occur. If the number of outages equals about 30% of the number of double differences of the complete observation scheme, it becomes more economical to compute the weight matrix starting from the corresponding variance-covariance matrix.

(3) Goad (1988) proposes to decorrelate the double difference measurements of one epoch (Cholesky decomposition of the contribution to the normal equation system at the epoch considered), which may lead to a more efficient updating of the normal equation system with the measurements of one epoch.

General Case: Different Linear Combinations (LC's) of L_1 and L_2 for Different Baselines

Imagine the following two scenarios:

Scenario 1: To measure a network you use both single and dual frequency GPS receivers (an example is given in Figure 3.1). When processing the data you would like to combine the L_1 data of the single (and dual) frequency instruments and the ionosphere-free linear combination (L_3, see Table 2.1) of L_1 and L_2 for the dual frequency receivers in one adjustment process.

Scenario 2: You have measured a network with a mixture of very short (a few kilometers) and long baselines. For the short baselines you introduce the L_1 and L_2 observations (because of the smaller noise) and for the long baselines L_3 (because of the ionosphere bias) into the parameter estimation program.