MECHANICS OF POROELASTIC MEDIA

SOLID MECHANICS AND ITS APPLICATIONS
Volume 35

Series Editor: G.M.L. GLADWELL
Solid Mechanics Division, Faculty of Engineering
University of Waterloo
Waterloo, Ontario, Canada N2L 3G1

Aims and Scope of the Series

The fundamental questions arising in mechanics are: *Why?*, *How?*, and *How much?* The aim of this series is to provide lucid accounts written by authoritative researchers giving vision and insight in answering these questions on the subject of mechanics as it relates to solids.

The scope of the series covers the entire spectrum of solid mechanics. Thus it includes the foundation of mechanics; variational formulations; computational mechanics; statics, kinematics and dynamics of rigid and elastic bodies; vibrations of solids and structures; dynamical systems and chaos; the theories of elasticity, plasticity and viscoelasticity; composite materials; rods, beams, shells and membranes; structural control and stability; soils, rocks and geomechanics; fracture; tribology; experimental mechanics; biomechanics and machine design.

The median level of presentation is the first year graduate student. Some texts are monographs defining the current state of the field; others are accessible to final year undergraduates; but essentially the emphasis is on readability and clarity.

For a list of related mechanics titles, see final pages.

Mechanics of Poroelastic Media

Edited by

A. P. S. SELVADURAI

Department of Civil Engineering and Applied Mechanics,
McGill University,
Montreal, Canada

SPRINGER-SCIENCE+BUSINESS MEDIA, B.V.

Library of Congress Cataloging-in-Publication Data

Mechanics of poroelastic media / edited by A.P.S. Selvadurai.
p. cm. -- (Solid mechanics and its applications ; v. 41)
Includes indexes.
ISBN 978-90-481-4513-3 ISBN 978-94-015-8698-6 (eBook)
DOI 10.1007/978-94-015-8698-6
1. Porous materials. 2. Elasticity. I. Selvadurai, A. P. S.
II. Series.
TA418.9.P6M45 1995
620.1'1692--dc20 95-24707

ISBN 978-90-481-4513-3

Printed on acid-free paper

Table of Contents

PREFACE

The mathematical theory of poroelasticity deals with the mechanical behaviour of an elastic porous medium which is either completely or partly filled with a pore fluid. The elastic porous phase can be either an assemblage of solid particles, a solid with a network of interconnected pores or a system of isolated pores. The pore fluids generally include air, water, oil, gas and biological fluids. During thermal, hydraulic, mechanical or electrochemical loading of a poroelastic medium, the various components of the multiphase system experience stresses and strains. The mutual interaction between the various phases of the poroelastic material will ultimately govern its thermo-hydraulic-mechanical behaviour.

The earliest recognition of the importance of the multiphase nature of porous geomaterials can be attributed to the pioneering studies by K.Terzaghi, published in 1925. In his development of the "*theory of effective stress*" for a geomaterial, Terzaghi postulated that when a saturated soil is subjected to external loading, this loading is carried partly by the porous soil skeleton and partly by the pore fluid. The ability of the pore fluid to be stressed during the application of an external loading is an important development in the understanding of the mechanical behaviour of saturated porous geomaterials. The second aspect of the influence of multiphase behaviour of geomaterials in their *time-dependent* behaviour was demonstrated by Terzaghi in the development of the classical theory of "*consolidation of soils*". This theory, which can be regarded as the origin of modern theories of poroelasticity, postulates that for soils with relatively low permeability, the load is initially borne by the pore fluid. With progress of time, the pore fluid pressures will dissipate and, at the termination of the consolidation process, the external loadings are borne entirely by the porous soil skeleton. The resulting time-dependent consolidation process results in deformations which are conventionally referred to as consolidation settlements.

The original developments by Terzaghi were primarily restricted to the examination of the one-dimensional consolidation behaviour of a saturated soil. These concepts were extended by M.A. Biot in a series of articles published between 1941 and 1956 to include three dimensional elastic behaviour of the soil skeleton, anisotropy of elastic behaviour and viscoelastic response of the soil skeleton. The theory of poroelasticity developed by Biot has, over the past five decades, received considerable attention within the geomechanics community. The theory has been used to develop elegant analytical solutions and computational schemes including finite element and boundary element solutions to problems in geomechanics associated with soil consolidation, offshore geotechnique, hydraulic fracturing for energy resources exploration and estimation of ground subsidence due to fluid withdrawal or heave due to fluid injection.

In recent years, the theory of poroelasticity has found extensive application in other areas, such as biomechanics of soft tissues, mechanics of bone, transport of multiphase fluids in porous media with special reference to applications in environmental geomechanics and energy resources recovery, geophysical applications with reference to the study of earthquake phenomena and in the study of advanced materials such as saturated micro-cellular foams and polymer composites. The renewed interest has resulted in the modification of Biot's classical theory to include more complex phenomena in the description of both the porous skeleton and the interstitial fluid.

The present volume contains a collection of papers which record the recent advances in the application of the theories of poroelasticity to problems of diverse interest. The contributions are grouped into primary topic areas covering constitutive modelling and analytical aspects, numerical modelling, applications in geomechanics, applications in biomechanics and dynamic effects. The contributions within these groups, however, do have several themes of common interest. The volume presents complete articles which document the versatility of theories of poroelasticity in addressing technologically important problems with significant contents in mathematics and mechanics. It is hoped that the volume will serve as a useful reference to students and researchers in geomechanics, biomechanics, applied mechanics and material science in illustrating the versatility of the classical theories of poroelasticity and the prospects for their further development.

A.P.S. Selvadurai
McGill University
Montréal, Canada

ACKNOWLEDGEMENTS

The papers contained in this volume were largely derived from the Mini-Symposium on "*Recent Developments in Poroelasticity*" organized by the Elasticity Committee of the Applied Mechanics Division of the American Society of Mechanical Engineers. The Symposium was a part of the Technical Program of the 1994 International Mechanical Engineering Congress held in Chicago, Illinois, U.S.A. As the organizer of the Symposium, the Editor would like to express his sincere thanks to Professor Millard F. Beatty (University of Nebraska), Chairman of the Elasticity Committee of the Applied Mechanics Division of the ASME, for his very generous support and encouragement in all aspects related to its scheduling within the activities of ASME. Thanks are also due to Professors Steve C. Cowin (The City College of CUNY) and Alex H.-D. Cheng (University of Delaware) for providing the names of additional participants which considerably enhanced the scope of both the Symposium and the Volume. The willingness of the authors to attend the Symposium and to contribute to the Volume is greatly appreciated both by the Editor and the Elasticity Committee of the Applied Mechanics Division of ASME. The authors are thanked for promptly attending to the various revisions associated with the contributions. The Editor would like to express his appreciation to Professor Graham M.L. Gladwell (University of Waterloo), Series Editor, for recommending the project as a Volume in the Kluwer Academic Publishers series '*Solid Mechanics and its Applications*'. The assistance provided by Mrs. Ann Bless, of the Department of Civil Engineering and Applied Mechanics at McGill University, in coordinating the correspondence is gratefully acknowledged.

Finally, the preparation of this Volume was greatly facilitated by the editorial assistance provided by Mrs. Sally Selvadurai, who completed the copy editing and compiled the Author and Keyword (Subject) Index.

SECTION 1

POROELASTICITY: CONSTITUTIVE MODELS AND ANALYTICAL ASPECTS

Moving and Stationary Dislocations in Poroelastic Solids and Applications to Aseismic Slip in the Earth's Crust

J. W. RUDNICKI
Departments of Civil Engineering,
Mechanical Engineering and Geological Sciences
Northwestern University
Evanston, IL 60208-3109

Abstract Results for the pore pressure induced by a plane strain shear dislocation that starts from rest, moves a finite distance at constant speed and stops demonstrate that coupling between deformation and diffusion causes a complex response even though the spatial distribution of slip is simple. A summary of recent solutions for stationary, instantaneous plane strain shear and opening dislocations and steadily moving shear dislocations demonstrates that coupling between deformation and diffusion is significant for locations near the dislocation edge and for short times. In addition, the response depends strongly on whether the plane of the dislocation is permeable or impermeable. Applications of these solutions to interpret water well level changes caused by aseismic slip (creep) in the Earth's crust are discussed.

1. Introduction

In portions of the earth's shallow crust that are infiltrated with ground water, the coupling of deformation with diffusion can significantly affect the mechanical response. In particular, this coupling has been identified as a factor in the propagation of hydraulic fractures (Ruina, 1978; Cleary, 1978; Detournay and Cheng, 1991a; Detournay *et al.*, 1990; Atkinson and Craster, 1991), time-dependent deformation following earthquake slip (Nur and Booker, 1972; Booker, 1974; Rice and Cleary, 1976; Rudnicki, 1986), propagating aseismic slip (creep) events (Rice and Cleary, 1976; Cleary, 1978; Rice and Simons, 1976; Simons, 1979; Rudnicki and Koutsibelas, 1991; Rudnicki, 1991). Detournay and Cheng (1991b) have recently summarized the effects of the coupling on a variety of rock mechanics problems, including borehole deformation and hydraulic fracture.

Solutions for dislocations in elastic solids provide elemental models for shear and tensile fractures. Even when the approximation of ideally elastic behavior outside of the

A. P. S. Selvadurai (ed.), Mechanics of Poroelastic Media, 3–22.

fracture is extreme, the solutions can provide insight into the magnitude and time scale of deformation. Furthermore, in applications to the earth's crust, material properties are often not known in sufficient detail to warrant more elaborate modeling. When more elaborate modeling is justified, dislocation solutions can provide the basis for efficient numerical modeling.

This paper reviews solutions for plane strain dislocations in poroelastic solids. The coupling of diffusion with deformation introduces a time dependence into the response in addition to any associated with imposition of the dislocation. The solutions reviewed include those for stationary shear and opening dislocations introduced instantaneously and for shear dislocations propagating steadily. In all cases, the plane containing the dislocation may be permeable or impermeable. In addition, we derive the solution for the pore pressure induced by a shear dislocation that moves non-steadily on a permeable plane. In particular, the dislocation starts from rest, moves at constant speed and stops.

The paper briefly summarizes application of the solutions to interpretation of water well level changes due to episodic creep events in the earth's crust. Creep events are slip episodes that occur too slowly to generate seismic waves. On some portions of large fault systems, like the San Andreas, creep can occur steadily at a rate comparable to the relative motion between tectonic plates (a few centimeters per year on the San Andreas). At other places, creep occurs intermittently in discrete events that appear to propagate laterally (parallel to the surface), toward the surface from depth or a combination of both. Because creep events occur relatively slowly, at least by comparison to propagation speeds of seismic wave or rupture velocities of earthquakes, the time scale of the associated deformation is comparable to that for ground water diffusion and the effects of coupling can be significant.

2. Governing Equations

In this section, we briefly summarize the governing equations for an isotropic poroelastic solid, beginning with the constitutive realtions. The presentation of the constitutive relations follows that of Rice and Cleary (1976). They improved upon Biot's (1941) formulation by exploiting the observation that in the limit of undrained response (no change in fluid mass content of material elements), the strains of the solid matrix are given by the usual elastic relation but with a different value of the Poisson's ratio.

For general deformations the strain tensor of the solid matrix ϵ_{ij} and the alteration of the fluid mass content m from some reference value m_0 depend on the total stress tensor σ_{ij} and the pore pressure p (measured from some ambient value) as follows:

$$\epsilon_{ij} = \frac{1}{2G}\left[\sigma_{ij} - \frac{\nu}{1+\nu}\sigma_{kk}\right] + p\delta_{ij}\,\frac{3(\nu_u - \nu)}{2GB(1+\nu)(1+\nu_u)} \tag{1}$$

$$m - m_0 = \frac{3\rho_0(\nu_u - \nu)}{2GB(1+\nu)(1+\nu_u)}\left[\sigma_{kk} + \frac{p}{3B}\right] \tag{2}$$

where ρ_0 is the density of the pore fluid (in the reference state), δ_{ij} is the Kronecker delta, the indicies (i, j) range over (1,2,3) and a repeated index denotes summation. In the limit of drained deformation, any alterations of pore fluid pressure are eliminated by fluid mass diffusion and, consequently, $p=0$. Thus, ν is identified as the drained Poisson's ratio and G is the shear modulus (which turns out to be the same for drained and undrained response). In the alternative limit of undrained deformation, the fluid mass in material elements remains constant, $m = m_0$, and from (2), $p=-B\,\sigma_{kk}/3$, where B is Skempton's coefficient. If this expression for the pore pressure is substituted into (1), the equation can be arranged in the form of the usual elasticity relation with ν_u replacing ν. Thus, ν_u is the undrained Poisson's ratio.

Rice and Cleary (1976) give the following expressions for B and ν_u:

$$B = \frac{1 - K/K_s'}{1 - K/K_s' + v_0 K/K_f - v_0 K/K_s''} \tag{3}$$

$$\nu_u = \frac{3\nu + B(1-2\nu)(1-K/K_s')}{3 - B(1-2\nu)(1-K/K_s')} \tag{4}$$

where K $(=2G(1+\nu)/3(1\text{-}2\nu)$) is the drained bulk modulus, K_f is the bulk modulus of the pore fluid, v_0 is porosity (in the reference state) and K_s'' and K_s' are bulk moduli that can be identified with the bulk modulus of the solid constituents if the pore space is interconnected and the solid constituent is locally isotropic and homogeneous (Nur and Byerlee, 1971; Rice and Cleary, 1976). From (3) and (4), it can be deduced that ν_u and B satisfy the following limits: $\nu \leq \nu_u \leq 1/2$ and $0 \leq B \leq 1$. In each case the upper bound occurs for separately incompressible solid and fluid constituents and the lower for a highly compressible pore fluid.

The final constitutive relation is Darcy's law which states that the fluid mass flux per unit area q_i is proportional to the gradient of the pore fluid pressure:

$$q_i = -\rho_0 \kappa \partial p/\partial x_i \tag{5}$$

where κ is a permeability more commonly expressed as k/μ where k has units of area and μ is the viscosity of the pore fluid.

The governing equations are completed by the strain-displacement relations

$$\epsilon_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right) \tag{6}$$

and equations expressing equilibrium of total stresses (in the absence of body forces) and fluid mass conservation

$$\frac{\partial \sigma_{ij}}{\partial x_i} = 0 \tag{7}$$

$$\frac{\partial q_i}{\partial x_i} + \frac{\partial m}{\partial t} = 0 \tag{8}$$

Rice and Simons (1976) showed that for plane strain deformation, the governing equations could be reduced to a set of four for the three in-plane stress components and pore pressure. Rudnicki (1987) and Rudnicki and Roeloffs (1990) followed this approach to obtain the dislocations solutions. Here, we illustrate an alternative approach and write the equations entirely in terms of the displacements and pore pressure. To this end, we invert the constitutive relations (1) and (2). The results are

$$\sigma_{ij} = 2G\epsilon_{ij} + (K - 2G/3)\epsilon_{kk}\delta_{ij} - \zeta p \delta_{ij} \tag{9}$$

$$m - m_0 = \rho_0 \zeta [\epsilon_{kk} + p/K_u B] \tag{10}$$

where $\zeta = 1 - K/K_s'$ and K_u is the undrained bulk modulus, $2G(1+\nu_u)/3(1-2\nu_u)$, which can also be expressed as

$$K_u = K/(1 - \zeta B) \tag{11}$$

Using the strain-displacement relations (6) in (9) and (10) and then substituting into equilibrium (7) and fluid mass conservation (8) yields

$$(K + G/3)\frac{\partial^2 u_k}{\partial x_k \partial x_j} + G\frac{\partial^2 u_j}{\partial x_k \partial x_k} - \zeta\frac{\partial p}{\partial x_j} = 0 \tag{12}$$

$$-\kappa\frac{\partial^2 p}{\partial x_k \partial x_k} + \zeta\frac{\partial}{\partial t}\left[\frac{\partial u_k}{\partial x_k} + \frac{p}{K_u B}\right] = 0 \tag{13}$$

A useful result is obtained by taking the divergence of (12):

$$\nabla^2[(K + 4G/3)e - \zeta p] = 0 \tag{14}$$

where $e = \partial u_k/\partial x_k$ and $\Delta^2(\ldots) = \partial^2(\ldots)/\partial x_k \partial x_k$ is the Laplacian. Using the result makes it possible to rewrite (13) as

$$c\nabla^2[e + \zeta p/K_u B] = \frac{\partial}{\partial t}[e + \zeta p/K_u B] \tag{15}$$

where c is the diffusivity:

$$c = \kappa\,\frac{K_u B(K + 4G/3)}{\zeta^2 K_u B + \zeta(K + 4G/3)} \tag{16}$$

or, in terms of the quantities used by Rice and Cleary (1976),

$$c = \kappa \frac{2GB^2(1+\nu_u)^2(1-\nu)}{9(\nu_u-\nu)(1-\nu_u)} \tag{17}$$

Comparison of (15) with (10) reveals that, as emphasized by Rice and Cleary (1976), the fluid mass content satisfies the homogeneous diffusion equation.

3. Formulation of Dislocation Problem

Consider plane strain deformation in the xy plane. As already mentioned, to solve the dislocation problem, Rudnicki (1987) and Rudnicki and Roeloffs (1990) followed the procedure of Rice and Simons (1976) in writing the governing equations in terms of the three in-plane stress components and the pore pressure. The conditions resulting from imposition of the dislocation were then expressed in terms of conditions on the stress and pore pressure on $y = 0$ and the resulting boundary value problem was solved in $y \geq 0$. Here, the equations (12-15) are in terms of displacements (where, for plane strain, the indicies range over (1,2) and $(x, y) = (x_1, x_2)$). Consequently, the boundary conditions can also be left in terms of displacements.

An instantaneous, plane strain shear dislocation at the origin in the xy plane corresponds to cutting the entire negative x axis, displacing the upper half ($y = 0^+$) to the right an amount $b/2$, the bottom ($y = 0^-$) to the left $b/2$ and then rejoining the two together. If this is done instantaneously at $t=0$, the condition on the x-direction displacement as $y=0$ is approached through positive values is

$$u_x(x,y=0^+) = (b/2)H(-x)H(t) \tag{18}$$

where H is the unit step function. Because u_y is anti-symmetric about $y = 0$ and is continuous, u_y vanishes on $y=0$:

$$u_y(x,y=0) = 0 \tag{19}$$

The pore pressure is also anti-symmetric about $y=0$. For a permeable fault the pore pressure is continous and, hence, must vanish on $y=$ 0:

$$p(x,y=0) = 0 \tag{20}$$

The gradient $\partial p/\partial y$ will not equal zero on $y = 0$. According to Darcy's law (5), the fluid mass flux in the y-direction is proportional to this gradient and, hence, the solution for the condition (20) is appropriate for slip on a plane that is permeable to the diffusing species.

There is, however, evidence from both laboratory and field studies that faults are often barriers to fluid flow (Wu *et al.*, 1975; Wang and Lin, 1978; Lippincott *et al.*, 1985). This can occur because the fault zone material contains significant amounts of clay or strongly comminuted material so as to make fluid flow difficult. Alternatively, a history of extensive shearing combined with repeated dissolution and precipitation can cause development of a strongly anisotropic fabric that allows fluid flow along the fault but prevents it across the fault (Rice, 1992; Byerlee, 1990). In either case, the limiting condition of a completely impermeable fault can be imposed by requiring

$$\partial p/\partial y = 0 \tag{21}$$

on $y = 0$. Because the pore pressure field must still be antisymmetric about $y = 0$ for the shear dislocation, the pore pressure on $y = 0$ is discontinous and approaches equal and opposite values as $y = 0$ is approached from above and below. This discontinuity is the limiting case of a severe gradient that would occur across a very narrow, but finite width, relatively impermeable fault zone.

In summary, the problem is reduced to solving equations (12) and (15) for u_x, u_y and p in $y \geq 0$, subject to the boundary conditions (18), (19) and one of (20) or (21) on $y = 0$.

The formulation here is for a shear dislocation introduced instantaneously. For an opening dislocation the roles of u_x and u_y in (18) and (19) would be interchanged. For a steadily moving dislocation, time can be eliminated by viewing the problem in a coordinate system that is translating with the steadily moving dislocation. As a consequence, the derivative $\partial(..)/\partial t$ in (15) can be replaced by $-V\partial/\partial x$. Thus, the boundary condition for a shear dislocation (in the moving coordinate system) would be (18) with the step function in time eliminated.

The solution can be obtained by the application of the Fourier transform on x and the Laplace transform on t. The Fourier transform of a function $\mathrm{f}(x,y,t)$ is defined by

$$\mathcal{F}[\mathrm{f}(x,y,t)] = \bar{\mathrm{f}}(\xi,y,t) = \int_{-\infty}^{\infty} \mathrm{f}(x,y,t)\exp(-\iota\xi x)dx \tag{22}$$

and the Laplace transform by

$$\mathcal{L}[\bar{\mathrm{f}}(\xi,y,t)] = F(\xi,y,s) = \int_{0}^{\infty} \bar{\mathrm{f}}(\xi,y,t)\exp(-st)dt \tag{23}$$

Application of the double transform to (14) and (15) yields

$$\left[\frac{d^2}{dy^2} - \xi^2\right] [(K + 4G/3)E - \zeta P] = 0 \tag{24}$$

$$\left[\frac{d^2}{dy^2} - (\xi^2 + s/c)\right] [(K_u - K)E + \zeta P] = 0 \tag{25}$$

where we have used (11) to eliminate the product $B\zeta$. The solutions for the doubly transformed solid dilation E and pore pressure P are

$$E = A(\xi,s)e^{-m(\xi)y} + B(\xi,s)e^{-n(\xi,s)y} \tag{26}$$

$$\zeta P = -(K_u - K)A(\xi,s)e^{-m(\xi)y} + (K+4G/3)B(\xi,s)\exp^{-n(\xi,s)y} \tag{27}$$

where

$$m^2(\xi) = \xi^2\,, \quad n^2(\xi,s) = \xi^2 + s/c \tag{28}$$

and the branches are chosen such that $\mathrm{Re}\{m(\xi)\} > 0$ and $\mathrm{Re}\{n(\xi,s)\} > 0$. An equation for the double transform of the x direction displacement U_x can be determined from using (26) and (27) in the double transform of the first of (12) ($j=1$). The solution of the resulting equation is

$$U_x = e^{-m(\xi)y}\left[C + A\frac{(K_u + G/3)}{G}\frac{\iota\xi y}{m(\xi)}\right] + \frac{\iota\xi B}{n^2(\xi,s) - \xi^2}e^{-n(\xi,s)y} \tag{29}$$

This expression and that for E (26) can be used to obtain an expression for dU_y/dy. Integration then yields

$$U_y = -\frac{e^{-m(\xi)y}}{m(\xi)}\left\{\frac{A}{G}\left[(K_u + 4G/3) + \frac{\xi^2 y(K_u + G/3)}{m(\xi)}\right] - \iota\xi C\right\} - \frac{Bn(\xi,s)}{n^2(\xi,s) - \xi^2}e^{-n(\xi,s)y} \tag{30}$$

Expressions for the doubly transformed stresses can be determined by differentiation with respect y and multiplication by $\iota\xi$. The conditions (18), (19), and either (20) or (21) can be used to determine A, B, and C. The physical stresses and displacements are obtained by inverting the transforms, which is the most difficult aspect of the solution. Details of the transform inversions for the stresses due to the instantaneous dislocations are given by Rudnicki (1987).

For the steadily moving dislocations only the Fourier transform is needed. The solution for the transformed pore pressure and displacements components is identical to that given here with s/c replaced by $-\iota\xi V/c$ in n. Details of the transform inversion for the steadily moving shear dislocations are given by Rudnicki and Roeloffs (1990).

The next two sections present some results for the pore pressure induced by the instantaneous and steadily moving dislocations. The following section derives results for the pore pressure induced by a shear dislocation on a permeable plane that moves with constant speed but starts from rest and stops.

4. Instantaneous Dislocation Solutions

The pore pressure caused by a plane strain shear dislocation introduced instantaneously at the origin in the xy plane is

$$p(x,y,t) = AbG\frac{y}{r^2}\left[1 - \exp(-\frac{r^2}{4ct})\right] \tag{31}$$

where $r = (x^2 + y^2)^{1/2}$ and A is the following combination of constants:

$$A = B(1+\nu_u)/3\pi(1-\nu_u) \tag{32}$$

This solution was first presented by Nur and Booker (1972). The solution for the stress field, assuming incompressible solid and fluid constituents, was derived by Booker (1974) using integral transforms and for arbitrarily compressible constituents by Rice and Cleary (1976) using complex variables. The pore pressure given by (1) vanishes on the plane $y = 0$ as required by the condition (20) but the gradient $\partial p/\partial y$ does not. Thus, as explained earlier, the solution is appropriate for slip on a plane that is permeable to the diffusing species.

Rudnicki (1986) presented the result for the pore pressure induced by a shear dislocation on an impermeable plane:

$$p(x,y,t) = AGb\left[\frac{y}{r^2}\mathrm{erf}(\frac{y}{\sqrt{4ct}}) + \frac{x}{r^2}\exp(-\frac{y^2}{4ct})\mathrm{D}(\frac{x}{\sqrt{4ct}})\right] \tag{33}$$

where erf(u) is the error function, D(u) is $2/\pi^{1/2}$ times Dawson's integral (Abramowitz and Stegun, 1964),

$$\mathrm{D}(u) = -\iota\exp(-u^2)\,\mathrm{erf}(\iota u) \tag{34}$$

and $\iota = \sqrt{(-1)}$. Equation (34) gives the result for $y \geq 0$; that for $y \leq 0$ is equal in magnitude and opposite in sign. Rudnicki (1987) used Fourier and Laplace transforms to derive the complete stress field.

Figure 1 plots the pore pressure predicted by (32) and (33), in the nondimensional form pr/AGb, against the nondimensional time $4ct/r^2$. In this form, the solution depends only on the angle θ and results are shown for $\theta = 0°$, 30° and 90°. For the permeable fault, the pore pressure is zero on $\theta = 0°$ and the result shown for the impermeable fault is that for $y = 0^+$. Notable differences between the solutions are the slower decay for the impermeable fault, the larger pore pressure changes at low angles for the impermeable fault and the initial increase with time of the pore pressure for the impermeable fault at angles less than 90°.

Rice and Cleary (1976) also give the solution for a plane strain opening dislocation. In this case the pore pressure field is symmetric about $y = 0$. Consequently, the pore pressure has a finite nonzero value on $y = 0$ but the gradient $\partial p/\partial y$, assumed to be continuous, vanishes on $y = 0$. Rudnicki (1987) gives the corresponding solution for an opening dislocation when the pore pressure is required to be zero on the x-axis. In this case, the gradient $\partial p/\partial y$ is discontinuous on $y = 0$ and takes on values equal in magnitude and opposite in sign as the x-axis is approached from above and below. For the opening dislocation, the condition $p = 0$ on $y = 0$ is a limiting idealization of a thin layer of high permeability in the x direction so that any alterations in pore pressure are eliminated by rapid fluid mass diffusion along this plane.

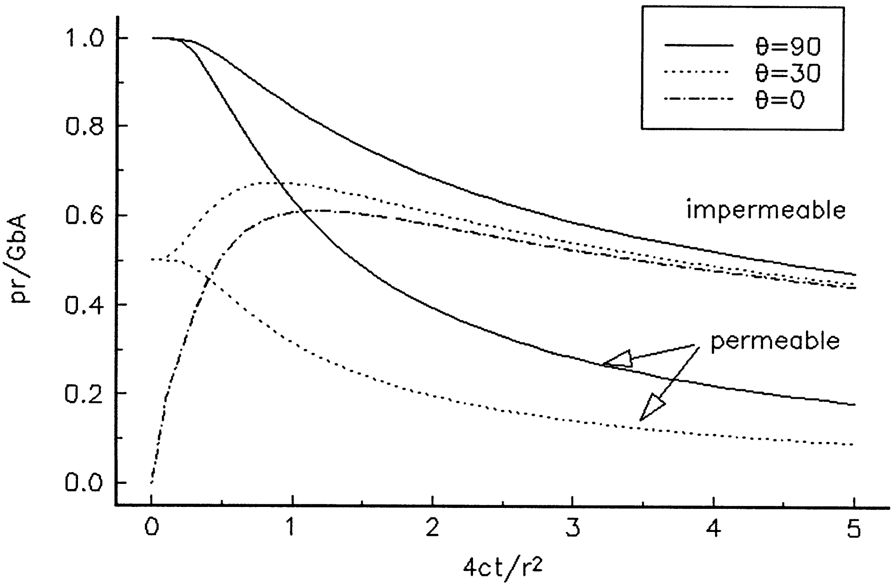

Figure 1. *Comparison of the pore pressure induced by sudden introduction of a shear dislocation on a permeable and an impermeable plane.*

The solutions for plane strain shear and opening dislocations can be written in compact form by introducing a complex displacement discontinuity (Burgers' vector), $b = b_x + \iota b_y$ and a complex position vector $z = x + \iota y$. The solution for a shear dislocation on a permeable plane and opening dislocation on an impermeable plane (continuous pore pressure and gradient) is

$$p(x,y,t) = -AG\,\mathrm{Im}\{bz^{-1}[1 - \exp(-r^2/4ct)]\} \tag{35}$$

where Im{...} stands for "the imaginary part of" {...}. The solution for a shear dislocation on an impermeable plane (discontinuous pore pressure) and opening dislocation on a permeable plane (discontinuous gradient) is

$$p(x,y,t) = -AG\,\mathrm{Im}\{bz^{-1}[1 - \mathrm{W}(x,y,t)]\} \tag{36}$$

where

$$\mathrm{W}(x,y,t) = \mathrm{erfc}[y/\sqrt{4ct}] + \exp(-r^2/4ct)\,\mathrm{erf}[\iota x/\sqrt{4ct}] \tag{37}$$

and erfc(u) is the complementary error function. Rudnicki (1987) has given the corresponding expressions for the complete stress fields.

Rudnicki *et al.* (1993) used the solutions (31) and (33) to interpret five slip-induced water level changes observed during January 1989 to July 1990 in a well 400 meters from the San Andreas fault near Parkfield, California. For the well studied and the frequency of the water level changes (see Roeloffs *et al.* (1989) for a more detailed discussion of the frequency dependence), the calculated pore pressure changes can be converted to water level changes by dividing by the weight density of water. By comparing the observed and calculated water level changes, Rudnicki *et al.* (1993) were able to infer the magnitude of the slip and the distance that the slip extended along the fault past the well. In three cases the inferred magnitude and extent of slip were consistent with that measured by creepmeters at the surface several hundred meters from the well. Discrepancies in the remaining two cases were interpreted to be the result of a depth variation of slip that was not included in the two dimensional model. Assuming the fault is impermeable required a higher diffusivity (0.15 m^2/s) than for a permeable fault (0.06 m^2/s) but otherwise had little effect.

The dislocation solutions themselves are an extreme and overly simple model of slip on actual fault systems, but, as demonstrated by Rudnicki *et al.* (1993), they are sometimes adequate to interpret the available data. More complex spatial distributions and time histories of slip can be constructed by superposition. Rudnicki and Hsu (1988) have presented results for a boxcar distribution of slip (uniform slip over a finite length) with a ramp increase of slip with time. They use the results to interpret a water level change observed by Lippincott *et al.* (1985) in a well near the Garlock fault. Lippincott *et al.* (1985) interpreted the water level changes by neglecting coupled deformation diffusion effects, using the ordinary elasticity solution for a steadily propagating distribution of slip, and assuming the pore pressure change is proportional to the negative of the mean stress change. That a satisfactory fit to the data could be achieved by either approach, with quite different slip distributions, suggests the necessity for knowing hydrologic conditions for detailed modeling.

5. Steady State Solutions

Solutions for dislocations moving steadily at a speed V can be constructed using the solutions for instantaneous dislocations and the superposition procedure described by Carslaw and Jaeger (1959). Cleary (1978) used this approach to obtain numerical results for opening and shear dislocations. Roeloffs and Rudnicki (1984/85) used the same approach to obtain the following result for the pore pressure induced by a shear dislocation moving steadily on a permeable plane:

$$p(X,Y) = AGb(Y/R^2)[1 - (VR/2c)\mathrm{K}_1(VR/2c)\exp(-VR/2c)] \tag{38}$$

where $\mathrm{K}_1(z)$ is the modified Bessel function of the second kind (Abramowitz and Stegun, 1964) and we have used upper case for the coordinates to emphasize that the origin moves with the dislocation. Roeloffs and Rudnicki (1984/85) used this solution to evaluate coupled deformation diffusion effects for propagating slip events and to re-examine a slip induced water level change reported by Johnson *et al.* (1973) and analyzed by Wesson (1981) using the ordinary elasticity solution for a steadily moving distribution of slip.

Rudnicki and Roeloffs (1990) used integral transforms to derive the complete stress and pore fluid pressure fields for shear dislocations moving on both permeable and